IJAME

Int. J. of Applied Mechanics and Engineering, 2024, vol.29, No.3, pp.150-165
DOI: 10.59441/ijame/188519

THE EFFECT OF TEMPERATURE ON THE VIBRATION BEHAVIOUR OF LAMINATED COMPOSITE PLATES

Mohamed Lamine Salhi and Mekki Mellas
Génie Civil et Hydraulique, Laboratoire de Recherche en Génie Civil (LRGC), ALGERIA

Houdayfa Ounis*
Centre de Développement des Energies Renouvelables, CDER, 16340 Algiers, ALGERIA
E-mail: Houdayfa.ounis@gmail.com

The present work is a contribution to the study of the effects of temperature on vibrations and stability of laminated composite plates using the finite element method. Thus, a DMQP/ml bending finite element with 4 nodes and 3 degrees of freedom based on the first order shear theory is extended to consider the effects of temperature on vibration and stability of laminated composite plates. The effect of the dependence of material properties on temperature as well as the effect of the of thermal stresses on the natural frequencies of laminated plates are studied simultaneously. A parametric study was carried out to highlight the effect of certain parameters on the vibration behaviour of the laminated plates. The study showed that in most cases the natural frequencies of vibration decrease with the increase in temperature. On the other hand, if the temperature inflicted on the plate coincides with the critical buckling temperature, the natural frequencies tend towards zero. Moreover, based on experimental data, this paper presents a study of the effect of temperature on the vibration behaviour of a laminated T300/5208 Graphite/Epoxy plate. The study showed that temperature significantly changes the properties of the materials as well as the vibration behaviour of the plate.

Key words: laminated composite plates, thermal buckling, vibration, finite-element method.

1. Introduction

Nowadays, composite technology has become trendy and has seen extraordinary expansion due to the remarkable advantages it offers, such as high resistance, high stiffness, long duration under fatigue, and low density [1, 2]. Therefore, their use is growing compared to traditional materials in basically all industrial domains, including aeronautics, aerospace, automotive, and civil engineering. Due to their intense use in various industrial fields, composite structures often work in very difficult environmental conditions, such as high temperatures, which can significantly influence their behaviour. In addition to the effect of temperature on the stability of composite structures, changes in temperature may affect their vibration behaviour. The first and obvious effect of temperature is the deterioration of material properties of the structure, which induces a change in the vibration behaviour of the composite structure [3, 4]. The second effect of temperature elevation is the development of thermal stresses, which can change the effective stiffness of the structure. This change in stiffness is not associated with the change in material properties but depends only on the state of stress [3-5].

In the following sections, we will present a review of the literature on important works on the effect of temperature on the vibration behaviour of composite laminated plates. This review will cover studies that have investigated the two effects of temperature, i.e., the effect of thermal stresses and the effect of temperature-dependent properties, on the vibration behaviour of composite laminated plates.

Tauchert [5] reported that one of the first works considering the vibration of plates with temperature-dependent properties is due to Fauconneau and Marangoni [6]. The authors studied the effect of linear temperature distribution on the vibration of simply supported isotropic plates using the Rayleigh-Ritz method.

---

* To whom correspondence should be addressed

Dhotarad and Ganesan [7, 8] examined the effect of temperature gradient on the dynamic free response of thin rectangular plates subjected to steady-state one-dimensional and two-dimensional temperature distributions. There are also other works in the literature on the effect of temperature gradient on the vibration behaviour of isotropic and orthotropic plates with varying thickness. Rao and Satyanarayana [9] studied the effect of temperature gradient on the fundamental frequencies of isotropic plates having a linearly varying thickness. Ganesan and Dhotarad [10] extended their work reported in [7] to the study of orthotropic plates having linearly varying thickness. Thereafter, Tomar and his colleagues [11-14] presented a series of works on the thermal effect on the vibration behaviour of rectangular and circular plates, as well as orthotropic plates having variable thicknesses and temperature-dependent properties. Adeniji-Fashola and Oyediran [15] investigated the thermal gradient effect on the vibration of prestressed rectangular plates. Gupta and his colleagues [16-19] presented several studies on the vibration of plates with linearly varying thickness in both directions. The authors considered the thermal gradient effect and the non-homogeneity to study the free vibration of orthotropic, viscoelastic, and trapezoidal plates.

Literature has given much attention to the effect of thermal stresses on the vibration behaviour of plates compared to the aforementioned effect. However, in this work, for the sake of brevity, only work on composite plates will be mentioned. Nevertheless, for further details on the vibration behaviour of isotropic plates under the effect of temperature, the reader may refer to the reviews [20-23].

It appears in the literature that work on the effect of temperature on the dynamic behaviour of composite plates began in the early 90s. Jeng-Shian *et al.* [24] studied the thermal impact on the vibration behaviour of thin laminated plates using the finite element method in 1992. The authors used an eight-nodded Lagrangian finite element. In the same year, Huang and Tauchert [25] presented an analytical study on the dynamic behaviour of doubly curved cross-ply laminated panels subjected to rapid heating. Noor and Burton [26] used an analytic three-dimensional elasticity solution for the free vibration and buckling of thermally stressed angle-ply composite plates. The authors considered different lamination and material parameters under arbitrary symmetric variation of the temperature in the thickness direction. In 1993, Bhimaraddi and Chandrashekhara [27] analysed the nonlinear vibrations, buckling, and post-buckling behaviour of heated angle-ply laminated plates using a parabolic shear deformation theory. Galea and White [4] investigated the effect of temperature on the dynamic response of thin CFRP plates experimentally as well as by using the finite element method. The authors presented a parametric study on cross-ply and angle-ply laminated plates under different boundary conditions and temperature distributions. The finite element method was also used by Chang and Shyong [28] to study the thermal impact on the dynamic behaviour of cylindrical laminated shells using a high order shear deformation theory.

Thereafter, Liu and Huang [29] investigated the linear and nonlinear free vibration of cross-ply laminated plates subjected to a change in temperature using a first-order shear deformation finite element. Lee and Lee [30] investigated the vibration behaviour of thermally post-buckled anisotropic plates using first-order shear deformable plate theory. Similar studies were also found on the vibration behaviour of piezo-laminated composite plates embedded with shape memory alloy fibres subjected to thermal loads [31, 32].

In 1999, Adams and Bert [33] examined the dynamic response of a simply supported symmetrically cross-ply laminated plate subjected to a thermal shock. Librescu and Lin [34] studied the static and dynamic nonlinear response of laminated plates and shells subjected to thermomechanical loading. Thereafter, Shen,Zheng and Huang [35] studied the dynamic response of shear deformable laminated plates exposed to thermomechanical loading and resting on a two-parameter elastic foundation. The authors used Reddy's high order shear deformable plate theory with the inclusion of the plate-foundation interaction and thermal effects due to temperature rise. Thereafter, Shiau and Kuo [36] investigated the free vibration of thermally buckled composite sandwich plates using a high precision triangular finite element. Singha,Ramachandra and Bandyopadhyay [37] studied the vibration behaviour of thermally stressed composite skew plates in the pre- and post-buckling states using a four-nodded finite element based on the first-order shear deformation theory.

In 2007, Vangipuram and Ganesan [38] studied the free and damped vibration behaviour of sandwich plates having stiff-layers and an isotropic viscoelastic core under thermal loads. The authors used the classical plate theory for the stiff-layers and the Reddy's theory for the core. The damped vibration was also considered by Jeyaraj *et al.* [39]. The authors studied the dynamic and acoustic behaviours of composite plates in a thermal environment. Thereafter, Matsunaga [40] developed a new high order shear deformation theory to analyse the fundamental frequencies and the critical buckling temperature of angle-ply plates. In 2009, Lal and Singh [41] examined the thermal effect on the vibration behaviour of laminated plates having random material properties and thermal expansion coefficients. Afterwards, Chen *et al.* [42] studied the vibration and stability behaviours of thermally stressed composite plates with temperature-dependent-proprieties. The study focused on the effect of material properties dependence on the temperature and the number of layers on the critical buckling temperature and the free vibration of cross-ply laminated plates.

For further insights into the influence of temperature on the behaviour of laminated composite materials, readers may refer to the research conducted by Garg and Chalak [43]. Indeed, the authors have presented comprehensive numerical results derived from published articles, highlighting the significance of geometric parameters, material properties, and hygrothermal fields on the behaviour of laminated composite and sandwich structures under varying environmental conditions. They conducted a thorough critical review of the existing literature to predict the behaviour of these structures in hygrothermal conditions, categorizing the studies into static, vibration, buckling, postbuckling, and miscellaneous areas (including transient, dynamic, and impact studies).

The present study aims to investigate the impact of temperature on the vibration characteristics of laminated composite plates. In contrast to previous research, this study aims to simultaneously examine two temperature-related factors, namely: the influence of thermal stresses and the effect of temperature-dependent material properties on the vibration behaviour of laminated plates. Therefore, a finite element based on the "Discrete Mindlin Quadrilateral Plate Multilayer" model is extended to investigate the vibration response of laminated plates with temperature-dependent materials. furthermore, a parametric study is conducted to assess the influence of temperature-dependent properties, thermal stresses, anisotropy, boundary conditions, and geometric parameters. Furthermore, to enhance the overall understanding of laminated composite materials' behaviour under realistic conditions, a study is conducted on analysing the impact of temperature on the vibration behaviour of the widely used T300/5208 Graphite/Epoxy composite. Through analysing the variations in structural response across different temperature levels, valuable insights can be gained regarding the composite's performance and behaviour.

2. Finite element formulation

2.1. Formulation of laminated composite plate:

As the study aims to study thin and thick laminated composite plates, the displacements field based on the first order shear deformation theory (FSDT) is given by:

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\beta_x(x,y), \\ v(x,y,z) &= v_0(x,y) + z\beta_y(x,y), \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{2.1}$$

where u_0, v_0, w_0 are the in-plane and transverse displacement components of the plate, respectively. β_x and β_y are the rotations of the normal to the mid-surface in two planes $x-z$ and $y-z$, respectively. The strain-displacement relations based on Reissner-Mindlin hypothesis is with the von Karman nonlinearity can be determined as:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 \\ \dfrac{\partial v}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 \\ \dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} \end{Bmatrix} + z \begin{Bmatrix} \dfrac{\partial \beta_x}{\partial x} \\ \dfrac{\partial \beta_y}{\partial y} \\ \dfrac{\partial \beta_x}{\partial y} + \dfrac{\partial \beta_y}{\partial x} \end{Bmatrix} = \underbrace{\{\varepsilon_{\mathrm{L}} + \varepsilon_{\mathrm{NL}}\}}_{\{\varepsilon\}} + z\{\kappa\},$$

$$\{\gamma\} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \beta_x + \dfrac{\partial w}{\partial x} \\ \beta_y + \dfrac{\partial w}{\partial y} \end{Bmatrix} \tag{2.2}$$

where $\{\varepsilon\},\{\kappa\},\{\gamma\}$ are the mid-plane strain, plate curvature and transverse shear strain, respectively.

As in this paper the material properties are assumed to be dependent on the temperature, The stress-strain relation of the composite laminated plate subjected to temperature rise ΔT is given by [44]:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11}(T) & \bar{Q}_{12}(T) & \bar{Q}_{16}(T) \\ \bar{Q}_{21}(T) & \bar{Q}_{22}(T) & \bar{Q}_{26}(T) \\ \bar{Q}_{31}(T) & \bar{Q}_{32}(T) & \bar{Q}_{36}(T) \end{bmatrix}_k \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_k - \Delta T \begin{bmatrix} \alpha_x(T) \\ \alpha_y(T) \\ \alpha_{xy}(T) \end{bmatrix}_k ,$$

$$\begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix}_k = \begin{bmatrix} K_{11}\bar{Q}_{44}(T) & K_{12}\bar{Q}_{45}(T) \\ K_{21}\bar{Q}_{54}(T) & K_{22}\bar{Q}_{55}(T) \end{bmatrix}_k \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}_k \tag{2.3}$$

where $\bar{Q}_{ij}$ are the transformed reduced stiffness and K_{ij} are the shear correction factors.

The forces and the moments resultants are related to the mid-surface strains and to the curvatures by:

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix} - \begin{bmatrix} N^{\mathrm{T}} \\ M^{\mathrm{T}} \end{bmatrix},$$

$$[T] = [C][\gamma]. \tag{2.4}$$

With [A], [B], [D] and $[C]$ the extensional, coupling and bending rigidity matrix, respectively, those can be defined by:

$$A_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[\bar{Q}_{ij}\right]_k \mathrm{d}z,\ \ B_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[\bar{Q}_{ij}\right]_k z\mathrm{d}z,$$
$$D_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[\bar{Q}_{ij}\right]_k z^2\mathrm{d}z,\ \ C_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[\bar{Q}_{ij}\right]_k \mathrm{d}z. \tag{2.5}$$

The thermal force and thermal moment resultants are defined as:

$$\left\{\mathrm{N}^{\mathrm{T}}, \mathrm{M}^{\mathrm{T}}\right\}^{\mathrm{T}} = \int_{-\frac{\mathrm{h}}{2}}^{\frac{\mathrm{h}}{2}} \left[\overset{q}{\mathrm{Q}}_{\mathrm{ij}}\right]\left[\Delta\mathrm{T}\{\alpha\}^{\mathrm{T}}\right](1, \mathrm{z})\mathrm{dz}. \tag{2.6}$$

The total potential energy of laminated plate subjected to thermal loading can be expressed as:

$$\Pi = \frac{1}{2}\int_v \Big(\{\varepsilon_L\}^T[A]\{\varepsilon_L\} + \{\varepsilon_L\}^T[B]\{\kappa\} + \{\kappa\}^T[B]\{\varepsilon_L\} + \{\kappa\}^T[D]\{\kappa\} + \{\gamma\}^T[C]\{\gamma\} +$$
$$-2\left(\{\varepsilon_L\}^T[N^T] + \{\kappa\}^T[M^T]\right) + \{\varepsilon_{NL}\}[N^T]\Big)dv. \tag{2.7}$$

The kinetic energy expression can be given by:

$$V = \frac{1}{2}\int_V \rho(\dot{u}^2 + \dot{v}^2 + \dot{w}^2)\,\mathrm{d}V\ . \tag{2.8}$$

A finite element called DQMP/ml *Discrete Mindlin Quadrilateral Plate Multilayer* is extended to the study of the effect of temperature on the vibration behaviour of laminated plates with temperature-depend-materials [45]. The DQMP/ml was developed by Sakami [46] to study the behaviour of multilayer plates. The present finite element is an extension to the multilayer case of the finite element DKMQ developed by Katili [47]. With a 4 nodes and 3 degrees of freedom, the DQMP/ml is based on a variational model, called Displacement Discrete Mindlin (DDM), developed by Ayad [48]. For further details, the reader may refer to the literature [45, 46, 49, 50].

2.1. Solution procedure

To study the effect of temperature on the vibration of laminated composite plates, one can proceed in two steps. The first step is to determine the critical buckling temperature of the laminated plate. This step helps determine the maximum temperature that can withstand the plate before losing its stability. Thus, it returns to solve the following generalized eigenvalue problem:

$$\left([K] + \lambda[K_G]\right)\{X\} = 0 \tag{2.9}$$

where $[K]\{X\}$ and $[K_G]$ are the global stiffness matrix, the global displacement vector and the global geometric matrix, respectively.

As the materials proprieties are considered to be dependent on temperature. The Eq.(2.9) is solved by the iterative numerical procedure with the following steps [51]:

(1) Assuming materials proprieties are constant, the thermal buckling load for the plates of temperature-independent material is obtained.
(2) Using the buckling temperature determined in the previous step, the temperature-dependent material properties may be decided, and the thermal buckling load is obtained again.
(3) Step (2) is repeated until the thermal buckling temperature converges.

After determining the critical buckling temperature, the second step comes to study the vibration behaviour of laminated plates with thermal stresses by resolving the following eigenvalue problem:

$$\left[\left([K]+[K_G]\right)-\omega^2[M]\right]\{X\}=0 . \tag{2.10}$$

3. Numerical results

In the following section, the material properties are taken as:

$$E_1/E_2 = 40,\ G_{12}=G_{13}=0.6E_2,\ G_{23}=0.5E_2,\ \nu_{12}=0.25 .$$

In addition, the fundamental frequencies are expressed in terms of dimensionless quantities, defined by:

$$\bar{\omega}=\omega a^2\sqrt{\frac{\rho}{E_2 h^2}} .$$

Table 1 shows the effect of the temperature elevation on the fundamental frequencies of a square laminated composite plate. A three-layered simply supported laminated composite plate with a $[0°/90°/0°]$ as stacking sequence was considered. The coefficients of thermal expansion were taken as follow: $\alpha_1 = 1.14\times 10^{-6}, \alpha_2 = 11.4\times 10^{-6}$.

Tab.1. Temperature and side-to-thickness ratio effect on the fundamental frequencies of a $[0°/90°/0°]$ simply supported laminated plate.

ΔT	a/h	present	Lal and Singh [41]	Shen *et al.*[35]
	20	17.604	17.523	17.483
0 C°	10	14.750	14.710	14.702
	5	10.198	10.245	10.263
	20	16.912	17.171	17.172
100 C°	10	14.547	14.636	14.597
	5	10.126	10.232	10.226
	20	16.190	16.812	16.853
200 C°	10	14.342	14.562	14.490
	5	10.053	10.219	10.188

The results were obtained for three side-to-thickness ratios ($a / h = 20$, 10 and 5) and three sets of thermal loading conditions (ΔT = 0 $C°$, 10 $C°$ and 200 $C°$). The Tab.1 gathers the results obtained by the present formulation and those found by a nine-nodded FSDT finite element [41] as well as with an analytical solution based on the HSDT [35]. It can be noticed that frequencies decrease with the increasing of the temperature for the three sets of side-to-thickness ratios.

3.1. Effect of the anisotropy ratio

Table 2 shows the fundamentals frequencies of square laminated composite plate subjected to temperature changes. A four-layered laminated plate with a $[0° / 90°]s$ as stacking sequence was considered. The plate is assumed to have a thermal expansion ratio $\alpha_2 = 11.4 \times 10^{-6}$ and four thermal expansion ratios $\alpha_1/\alpha_2 = -0.05, 0.1, 0.2$ and 0.3.

The obtained results were compared with those found by Liu and Huang [29] with an FSDT isoparametric eight-nodded finite element for three state of temperature ($\Delta T = -50\,C°$, 0 $C°$ and 50 $C°$). From table 2, it can be seen that the fundamentals frequencies increase and decrease with the temperature depending on the value of the thermal expansion ratio α_1/α_2 .

From Tabs 1 and 2, It can be seen that the results found by the present finite element are in very good agreement with those from the literature.

Tab.2. Effect of the temperature and the ratio of the coefficients of thermal expansion on the natural frequencies of a laminated plate.

α_1/α_2		-50 C°	0 C°	50 C°
	Present	15.135	15.149	15.164
-0.05	Liu and Huang [29]	15.136	15.150	15.164
	Present	15.247	15.149	15.051
0.1	Liu and Huang [29]	15.247	15.150	15.052
	Present	15.321	15.149	14.976
0.2	Liu and Huang [29]	15.320	15.150	14.978
	Present	15.395	15.149	14.900
0.3	Liu and Huang [29]	15.394	15.150	14.902

3.2. Critical buckling temperature with temperature dependence (DT)

As aforementioned, a part of the study consists of finding the critical buckling temperature before starting the calculation of the fundamental frequencies of the plate. Therefore, it is necessary to verify the accuracy of the used element finite element in determining the critical buckling temperature with/ without temperature-dependent-properties. For so doing, a square simply supported $[0° / 90°]s$ laminated plate was considered with side-to-thickness ratio $a / h = 30$. The material proprieties were considered to be in linear function of temperature as follow [52]:

$$E_1(T) = E_{10}(1 + E_{11}T), \quad E_2(T) = E_{20}(1 + E_{21}T), \; \alpha_2(T) = \alpha_{20}(1 + \alpha_{21}T), \; \alpha_1(T) = \alpha_{10}(1 + \alpha_{11}T),$$

$$G_{12}(T) = G_{120}(1 + G_{121}T), \; G_{13}(T) = G_{130}(1 + G_{131}T), \; G_{23}(T) = G_{230}(1 + G_{231}T),$$

where

$$E_{10}/E_{20}=40,\ \ G_{120}/E_{20}=G_{130}/E_{20}=0.5,\ \ G_{230}/E_{20}=0.2,$$

$$\nu=0.25,\ \alpha_{10}=10^{-6}\left(°C^{-1}\right),\ \ \alpha_{20}=10^{-5}\left(°C^{-1}\right),\ \ \alpha_{11}=\alpha_{21}=0.5\times10^{-3},$$

$$E_{11}=-0.5\times10^{-3},\ \ E_{21}=G_{121}=G_{131}=G_{231}=-0.2\times10^{-3}.$$

Table 3 gathers the results obtained by the present element and those found by an Hermitian Layer-wise finite element [52] as well as the HSDT of Shen [53]. From the table, it can be seen that the obtained results are in excellent agreement with those from the literature in case of temperature- dependent-proprieties (TDP) or temperature-independent-proprieties (TIP).

Tab.3. Critical buckling temperature for a simply supported laminate plate with / without temperature dependence.

		TIP	TDP	TIP/TDP
$[0/90]_s$	Present	*0.667*	*0.523*	*1.275*
	HSDT [53]	*0.667*	*0.525*	*1.270*
	Layer-wise [52]	*0.640*	*0.496*	*1.290*

3.3. Parametric study

The purpose of the parametric study is to consider, simultaneously, two effects generated by the change in temperature on the fundamental frequencies of laminated plates, namely:

1) The effect of temperature-dependent-proprieties (TDP).
2) The effect of thermal stresses.

Unless indicated, we consider the following:

- The material proprieties and their changing function with the temperature are considered.
- The fundamental frequencies are expressed in terms of dimensionless quantities, defined by:

$$\bar{\omega}=\omega a^2\sqrt{\frac{\rho}{E_2h^2}}\ .$$

3.3.1. Effect of the temperature-dependent-proprieties

To study the effect of the temperature-dependent-proprieties, a square cross-ply laminated plate with a $[0°/90°]s$ as stacking sequence was considered. Three types of boundary conditions (SS, CC, CS) and a side-to-thickness ($a/h=30$) were taken. In this study, several types of thermal load defined according to the critical buckling temperature were applied, namely: T_0/T_{cr} = *0*, *0.25*, *0.5*, *0.75* and *1*.

Table 4 shows the effect of the temperature on the fundamental frequencies with temperature-dependent-proprieties (TDP) and without (TIP). From Tab.4, it is well seen that the fundamental frequencies decrease with the increasing in the temperature for the three boundary conditions. It is also noticed that the consideration of the dependence of temperature reduces the frequencies. This is probably because of the reduction in material properties which induces a reduction in stiffness.

It is very interesting to note that if the applied temperature is equal to the critical buckling temperature of the plate, the fundamental frequencies tend to zero. Indeed, this was observed for the case of sandwich plates [34, 36], and for the case of isotropic plates [54, 55]).

Table 4. Effects of temperature on the natural frequencies of laminated composite plates.

B.C.		T_0 / T_{cr}				
		0	*0.25*	*0.5*	*0.75*	*1*
SS	TIP	*17.7533*	*15.3762*	*12.5534*	*8.8776*	*0*
	TDP	*17.7533*	*15.3293*	*12.4418*	*8.7191*	*0.049*
CC	TIP	*32.7673*	*28.7123*	*23.7985*	*17.1787*	*0*
	TDP	*32.7673*	*28.1296*	*22.5678*	*15.5566*	*0.13796*
CS	TIP	*29.7318*	*26.8093*	*23.5074*	*19.6379*	*0*
	TDP	*29.7318*	*26.2821*	*22.3152*	*17.6898*	*0.1192*

3.3.2. Effect of the aspect ratio (a/b)

In this section, the effect of the aspect ratio (a / b) on the fundamental frequencies of laminated plates subjected to temperature elevation was considered. Two types of stratification $\left[0_2^\circ / 90_2^\circ\right]s$ et $\left[90_2^\circ / 0_2^\circ\right]s$ were taken with a side-to-thickness ratio $a / h = 10$.

In the solution procedure previously described, it was established to first calculate the critical buckling temperature. However, it should be noted that the relations describing the variation of the material proprieties with the temperature are not suitable for high values. To better explain, let’s take the relation of the longitudinal Young modulus $E_1(T)$:

$$E_1(T) = E_{10}(1 + \overbrace{E_{11}T}^{\eta}) .$$

From the relation, it can be observed that if $\eta \leq -1$ we obtain $E_1(T) \leq 0$ which is irrational. Thus, if the critical buckling temperature begets an $E_1(T) \leq 0$, then the solution diverges. As our concern is, only, not to exceed the critical buckling temperature, we decided to express it in the following form [42]:

$$T^* = T_{cr}\alpha_2 10^4 .$$

Figure 1 shows the effect of aspect ratio and the temperature on the fundamental frequencies of a simply supported cross-ply laminated plates. Three types of thermal loading were considered, namely: $T/T^* = 0,0.5,1$. From the Fig.1, it can be seen that the fundamental frequencies decrease with the increasing of the aspect ratio (a/b) and the temperature for both stacking sequences. It is also observed that the $\left[0_2^\circ / 90_2^\circ\right]s$ gives higher natural frequencies. It can be explained by the fact that the $\left[0_2^\circ / 90_2^\circ\right]s$ s has higher flexural rigidity than the $\left[90_2^\circ / 0_2^\circ\right]s$ in the direction of elongation of the plate.

3.3.3. Effects of temperature on the natural frequencies of vibration of laminated plates in T300 / 5208

In the previously reported studies, it was assumed that the material properties are a linear function of temperature. Besides, this suggestion was considered as such by many authors [23, 42, 51, 52, 56]. Although, it is well known that most of the components of a composite structure do not present the same behaviour with the temperature [57, 58]. Therefore, in order to be closer to practical cases, it is better to study structures with a well-defined material.

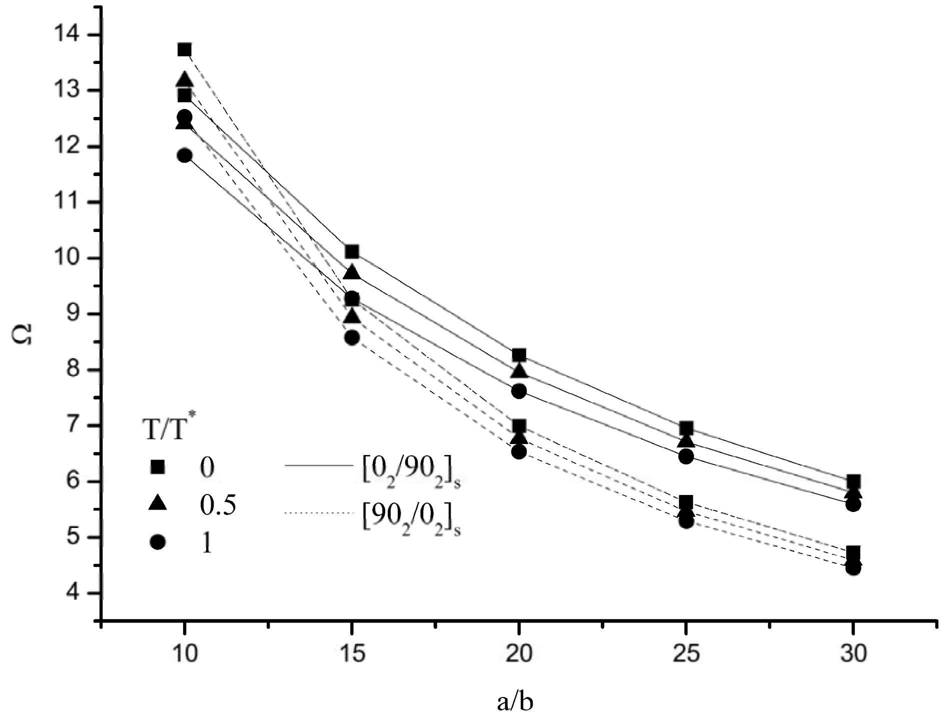

Fig.1. Effects of aspect ratio (a/b) and temperature on the natural frequencies of vibration of simply supported plates (SS).

There have been several experimental studies in the literature on the temperature-dependent-proprieties of the T300/5208 graphite/epoxy [57, 58]. The aim of these studies was to determine the elastic constants, ultimate strengths and thermal expansion behaviour of T300/5208 graphite/epoxy laminates at low and elevated temperatures. In this section, the work of Hyer *et al.* [59] is considered, where they studied specimens made from Thornel (Union Carbide) T300 graphite fibres and Narmco 5208 epoxy resin. The elastic constants and thermal expansion were determined in a temperature range from *116 K* (*-157.15°C*) to *394 K* (*120.85°C*) with a room temperature of *301 K* (*27.85°C*). The Fig.2 shows the results of Hyer *et al.* [59] in graphical form, where experimental data points and least square fit were presented for the following properties: E_1, E_2, G_{12} and α_1. From the Fig.2, it is well seen that the properties do not exhibit the same behaviour with the temperature. The coefficients of the least squares fit are presented in Tab.5.

In the following, the effect of temperature on the fundamental frequencies of T300/5208 graphite/epoxy laminated plate using Hyer *et al.* [59] coefficients were considered. A square laminated plate with a side-to-thickness ratio $(a/h = 10)$ and $[\pm\theta]_s$ stacking sequence was considered. Based on the results of Hyer *et al.* [59], five (5) types of thermal loadings were chosen, namely: *133.15 K* (*-140°C*), *213.15 K* (*-60°C*), *301 K* (*27.85°C*), *333.15 K* (*60°C*) and *368.15 K* (*95°C*). The material properties assumed independent temperature are given as follows: $\rho = 1.58\ \left(Kg/m^3\right)$, $\nu = 0.28$, $G_{23} = 3.35$ (GPa), $\alpha_2 = 26.5\ \left(10^{-6}/°C\right)$.

Figure 3 shows the effect of the temperature on the fundamental frequencies of simply supported T300/5208 graphite/epoxy laminated plate for different orientation angles. From the Fig.3, it is seen that the fundamental frequencies decrease with increasing of the temperature until the ambient temperature. However, when exceeding room temperature, a slight decrease in frequencies with temperature is noted for the angles *0°* to *60°*, whereas with *75°* and *90°* the frequencies increase with increasing of the temperature. This may be due to the negative value of the axial thermal expansion of the T300/5208 in this range of temperature. With a negative coefficient of thermal expansion, the plate generates tensile forces rather than compression, which increases the rigidity of the plate and thus increases the fundamental frequencies [45, 60-62].

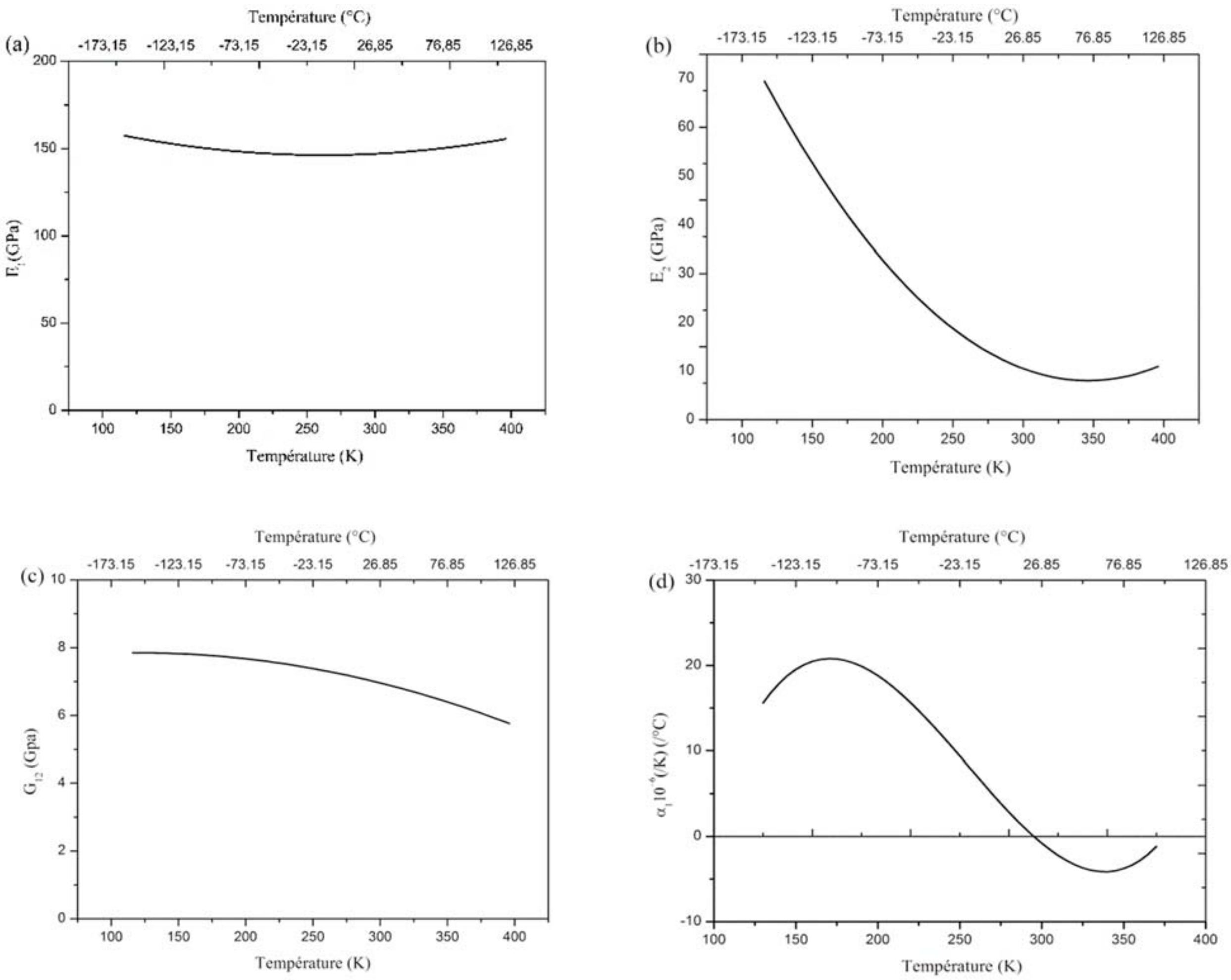

Fig.2. Elastic properties and coefficient of thermal expansion of T300 / 5208 as a function of temperature [59]. a) Longitudinal modulus of elasticity (E_1), b) Transverse modulus of elasticity (E_2), c) Shear modulus (G_{12}), d) coefficient of longitudinal expansion (α_1).

Tab.5. Temperature dependence coefficients for each property in Celsius.

Propriety*		C_0	C_1	C_2	C_3
E_1 (GPa)	K	182	-0.272	5.18×10^{-4}	0
	$°C$	146.35	0.0109		
E_2 (GPa)	K	147	-0.169	1.16×10^{-3}	0
	$°C$	14.21	-0.803		
G_{12} (GPa)	K	7.46	0.021	-2.73×10^{-5}	0
	$°C$	3.64	6.52×10^{-3}		
$\alpha_1(10^{-6}\ /°C)$	K	-110	. -0.212 .	0.586×10^{-3}	0.686×10^{-5}
	$°C$	4.275	18	-0.810×10^{-2}	0.106×10^{-4}

* $P = C_0 + C_1T + C_2T^2 + C_3T^3$ where P is the property of interest and (T) is temperature

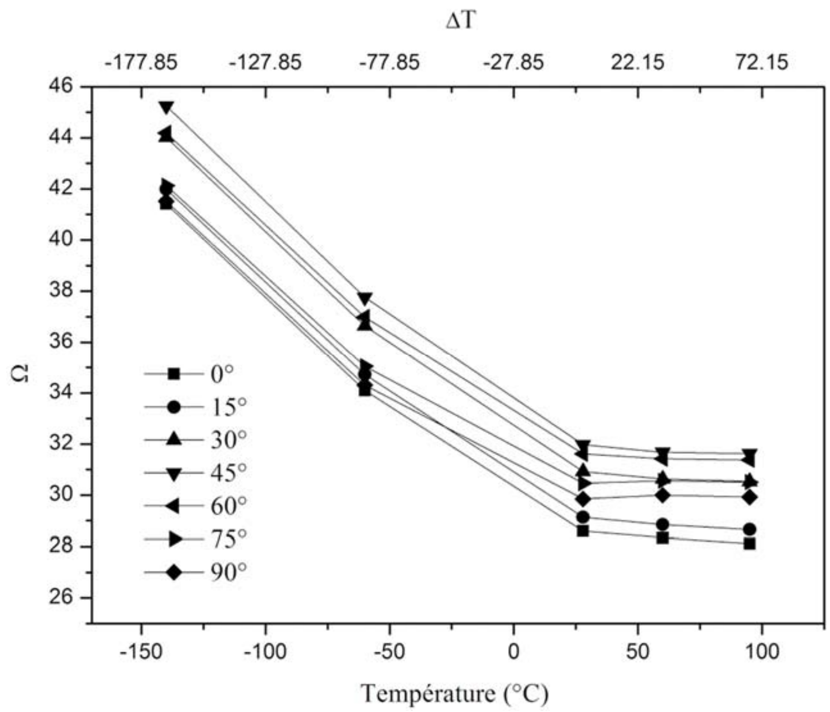

Fig.3. Effect of temperature and orientation angle on the natural frequencies of vibration of a plate laminated in T300 / 5208 simply supported (SS).

4. Conclusion

In this paper, a finite element based on the first order shear deformation theory, was used towards the study of the effect of temperature on the vibration and the stability of thick laminated plates under a uniform temperature distribution. Two effects are considered simultaneously, namely: The effect of the dependence of material properties on temperature rise, and the effect of thermal stresses. The comparison of the results obtained with reference solutions, determined analytically, and those obtained by other finite element models available in the literature, showed the performance and precision of the proposed numerical model. In addition, the parametric study has shown, the effect of different parameters on the natural frequencies of a laminated plate, such as the thickness ratio, the boundary conditions, the aspect ratio, the anisotropy ratio and the stacking sequence. The study has shown that in most cases the natural frequencies of vibration decrease with the increase in temperature. On the other hand, if the temperature inflicted on the plate coincides with the critical buckling temperature, the natural frequencies tend towards zero. In addition, a study was presented on the behaviour of laminated composite plates made of Graphite / Epoxy T300 / 5208. Based on the experimental work of Hyer *et al*. [59], the study showed that in the general case the natural frequencies of vibration decrease with the increase of the temperature. However, with some angles of orientation, if the material has a negative coefficient of thermal expansion, the natural frequencies of vibration increase with the increase in temperature.

Acknowledgements

The authors gratefully acknowledge the discussions with Professor Adel Benchabane to bring this manuscript in the present form.

Nomenclature

a,b	– plate dimensions along x and y axes respectively.
$[A],[D],[B],[C]$	– extensional, bending, coupling and transverse shear matrixes respectively.
$°C$	– temperature in Celsius.
E_1,E_2	– longitudinal and transverse modulus of elasticity.
G_{12},G_{13},G_{23}	– shear moduli of a lamina with respect to 1, 2 and 3 axes.
K	– temperature in Kelvin.
$[K],[K_G],\{X\}$	– global stiffness matrix, global geometry matrix and global displacement vector.
K_{ij}	– shear correction factors.
$[M]$	– global mass matrix.
$\bar{Q}_{ij}$	– transformed reduced stiffness.
T_{cr}	– critical buckling temperature.
T_0	– thermal load.
ΔT	– temperature rise.
u_0,v_0,w_0	– membrane and out of plane displacements of the plate.
V	– kinetic energy expression.
x,y,z	– system of coordinate axes.
υ	– Poisson ratio.
α_1,α_2	– coefficient of longitudinal and transverse expansion.
α_x, α_y, α_{xy}	– transformed thermal expansion coefficient.
β_x,β_y	– rotations of the normal to the mid-surface in two planes x-z and y-z.
$\{\varepsilon\},\{\kappa\},\{\gamma\}$	– mid-plane strain, plate curvature and transverse shear strain.
λ	– eigen value.
Π	– total potential energy of laminated plate.
ρ	– density.
ω	– natural frequency.
Ω	– fundamental frequencies.

References

[1] Brigante D. (2013): *New Composite Materials: Selection, Design, and Application*.– Springer International Publishing.

[2] Ounis H., Tati A. and Benchabane A. (2014): *Thermal buckling behaviour of laminated composite plates: a finite-element study*.– *Front. Mech. Eng.*, vol.9, No.1, pp.41-49, doi. 10.1007/s11465-014-0284-z

[3] Singer J., Arbocz J. and Weller T. (2002): *Buckling Experiments, Shells, Built-up Structures, Composites and Additional Topics*.– Wiley.

[4] Galea S.C.P. and White R.G. (1993): *The effect of temperature on the natural frequencies and acoustically induced strains in CFRP plates*.– Journal of Sound and Vibration, vol.164, No.3, pp.399-424, doi. 10.1006/jsvi.1993.1225.

[5] Tauchert T. (2014): *Plates with Temperature-Dependent Properties*.– Springer Netherlands.

[6] Fauconneau G. and Marangoni R. D. (1970): *Effect of a thermal gradient on the natural frequencies of a rectangular plate*.– International Journal of Mechanical Sciences, vol.12, No.2, pp.113-122, doi. 10.1016/0020-7403(70)90011-1.

[7] Dhotarad M.S. and Ganesan N. (1978): *Vibration analysis of a rectangular plate subjected to a thermal gradient*.– Journal of Sound and Vibration, vol.60, No.4, pp.481-497, doi. 10.1016/S0022-460X(78)80087-X.

[8] Dhotarad M.S. and Ganesan N. (1979): *Influence of thermal gradient on natural frequency of rectangular plate vibration*.– Nucl. Eng. Des., vol.52, No.1, pp.71-81, doi. 10.1016/0029-5493(79)90009-8.

[9] Rao C.K. and Satyanarayana B. (1975): *Effect of thermal gradient on frequencies of tapered rectangular plates*.– AIAA Journal, vol.13, No.8, pp.1123-1126, doi. 10.2514/3.6965.

[10] Ganesan N. and Dhotarad M.S. (1979): *Influence of a thermal gradient on the natural frequencies of tapered orthotropic plates*.– Journal of Sound and Vibration, vol.66, No.4, pp.621-625, doi. 10.1016/0022-460X(79)90706-5.

[11] Tomar J.S. and Gupta A.K. (1983): *Thermal effect on frequencies of an orthotropic rectangular plate of linearly varying thickness*.– Journal of Sound and Vibration, vol.90, No.3, pp.325-331, doi. 10.1016/0022-460X(83)90715-0.

[12] Tomar J.S. and Gupta A.K. (1984): *Thermal Effect on Axisymmetric Vibrations of an Orthotropic Circular Plate of Variable Thickness*.– AIAA Journal, vol.22, No.7, pp.1015-1017, doi. 10.2514/3.48544.

[13] Tomar J.S. and Gupta A.K. (1985): *Effect of thermal gradient on frequencies of an orthotropic rectangular plate whose thickness varies in two directions*.– Journal of Sound and Vibration, vol.98, No.2, pp.257-262, doi. 10.1016/0022-460X(85)90389-X.

[14] Tomar J.S. and Tewari V.S. (1981): *Effect of thermal gradient on frequencies of a circular plate of linearly varying thickness*.– J. Non-Equilib. Thermodyn., vol.6, No.2, pp.115-122.

[15] Adeniji-Fashola A.A. and Oyediran A.A. (1988): *Thermal gradient effects on the vibration of prestressed rectangular plates*.– Acta Mechanica, vol.74, No.1-4, pp.235-248, doi. 10.1007/bf01194357.

[16] Gupta A.K., Johri T. and Vats R.P. (2007): *Thermal effect on vibration of non-homogeneous orthotropic rectangular plate having bi-directional parabolically varying thickness*.– Proceedings of the World Congress on Engineering and Computer Science "WCECS" (San Francisco, USA, 24-26 october 2007, 2007), [insert City of Publication],[insert 2007 of Publication].

[17] Gupta A.K. and Kaur H. (2008): *Study of the effect of thermal gradient on free vibration of clamped visco-elastic rectangular plates with linearly thickness variation in both directions*.– Meccanica, vol.43, No.4, pp.449-458, doi. 10.1007/s11012-008-9110-1.

[18] Gupta A.K. and Sharma P. (2010): *Study the thermal gradient effect on frequencies of a trapezoidal plate of linearly varying thickness*.– Applied Mathematics, vol.1, No.05, pp.357, doi. 10.4236/am.2010.15047.

[19] Gupta A.K., Tripti Johri and Vats R.P. (2010): *Study of thermal gradient effect on vibrations of a non-homogeneous orthotropic rectangular plate having bi-direction linearly thickness variations*.– Meccanica, vol.45, No.3, pp.393-400, doi. 10.1007/s11012-009-9258-3.

[20] Bargmann H. (1974): *Recent developments in the field of thermally induced waves and vibrations*.– Nucl. Eng. Des., vol.27, No.3, pp.372-385, doi. 10.1016/0029-5493(74)90181-2.

[21] Tauchert T.R. (1991): *Thermally Induced flexure, buckling, and vibration of plates*.– Applied Mechanics Reviews, vol.44, No.8, pp.347-360, doi. 10.1115/1.3119508.

[22] Noda N. (1991): *Thermal stresses in materials with temperature-dependent properties*.– Applied Mechanics Reviews, vol.44, No.9, pp.383-397, doi. 10.1007/978-94-015-8200-1_2.

[23] Hetnarski R.B. (2014): *Encyclopedia of Thermal Stresses*.– Springer Reference.

[24] Jeng-Shian C., Jiunn-Hsiung W. and Tseng-Zong T. (1992): *Thermally induced vibration of thin laminated plates by finite element method*.– Comput. Struct., vol.42, No.1, pp.117-128, doi. 10.1016/0045-7949(92)90541-7.

[25] Huang N.N. and Tauchert T.R. (1992): *Thermally induced vibration of doubly curved cross-ply laminated panels*.– Journal of Sound and Vibration, vol.154, No.3, pp.485-494, doi. 10.1016/0022-460X(92)90781-R.

[26] Noor A.K. and Burton W.S. (1992): *Three-dimensional solutions for the free vibrations and buckling of thermally stressed multilayered angle-ply composite plates*.– Journal of Applied Mechanics, vol.59, No.4, pp.868-877, doi. 10.1115/1.2894055.

[27] Bhimaraddi A. and Chandrashekhara K. (1993): *Nonlinear vibrations of heated antisymmetric angle-ply laminated plates*.– International Journal of Solids and Structures, vol.30, No.9, pp.1255-1268, doi. 10.1016/0020-7683(93)90015-Y.

[28] Chang J.-S. and Shyong J.-W. (1994): *Thermally induced vibration of laminated circular cylindrical shell panels*.– Compos. Sci. Technol., vol.51, No.3, pp.419-427, doi. 10.1016/0266-3538(94)90110-4.

[29] Liu C.-F. and Huang C.-H. (1996): *Free vibration of composite laminated plates subjected to temperature changes*.– Comput. Struct., vol.60, No.1, pp.95-101, doi. 10.1016/0045-7949(95)00358-4.

[30] Lee D.-M. and Lee I. (1997): *Vibration behaviours of thermally postbuckled anisotropic plates using first-order shear deformable plate theory*.– Comput. Struct., vol.63, No.3, pp.371-378, doi. 10.1016/S0045-7949(96)00378-1.

[31] Oh I. K., Han J.H. and Lee I. (2000): *Postbuckling and vibration characteristics of piezolaminated composite plate subject to thermo-piezoelectric loads*.– Journal of Sound and Vibration, vol.233, No.1, pp.19-40, doi. 10.1006/jsvi.1999.2788.

[32] Park J.-S., Kim J.-H. and Moon S.-H. (2004): *Vibration of thermally post-buckled composite plates embedded with shape memory alloy fibers*.– Compos. Struct., vol.63, No.2, pp.179-188, doi. 10.1016/S0263-8223(03)00146-6.

[33] Adams R. J. and Bert C. W. (1999): *Thermoelastic vibrations of a laminated rectangular plate subjected to a thermal shock*.– J. Therm. Stresses, vol.22, No.9, pp.875-895, doi. 10.1080/014957399280607.

[34] Librescu L. and Lin W. (1999): *Non-linear response of laminated plates and shells to thermomechanical loading: Implications of violation of interlaminar shear traction continuity requirement*.– International Journal of Solids and Structures, vol.36, No.27, pp.4111-4147, doi. 10.1016/S0020-7683(98)00185-1.

[35] Shen H.-S., Zheng J.J. and Huang X.L. (2003): *Dynamic response of shear deformable laminated plates under thermomechanical loading and resting on elastic foundations*.– Compos. Struct., vol.60, No.1, pp.57-66, doi. 10.1016/S0263-8223(02)00295-7.

[36] Shiau L.C. and Kuo S.Y. (2005): *Free vibration of thermally buckled composite sandwich plates*.– Journal of Vibration and Acoustics, vol.128, No.1, pp.1-7, doi. 10.1115/1.2149388.

[37] Singha M.K., Ramachandra L.S. and Bandyopadhyay J.N. (2006): *Vibration behaviour of thermally stressed composite skew plate*.– Journal of Sound and Vibration, vol.296, No.4-5, pp.1093-1102, doi. 10.1016/j.jsv.2006.01.070.

[38] Vangipuram P. and Ganesan N. (2007): *Buckling and vibration of rectangular composite viscoelastic sandwich plates under thermal loads*.– Compos. Struct., vol.77, No.4, pp.419-429, doi. 10.1016/j.compstruct.2005.07.012.

[39] Jeyaraj P., Ganesan N. and Padmanabhan C. (2009): *Vibration and acoustic response of a composite plate with inherent material damping in a thermal environment*.– Journal of Sound and Vibration, vol.320, No.1-2, pp.322-338, doi. 10.1016/j.jsv.2008.08.013.

[40] Matsunaga H. (2007): *Free vibration and stability of angle-ply laminated composite and sandwich plates under thermal loading*.– Compos. Struct., vol.77, No.2, pp.249-262, doi. 10.1016/j.compstruct.2005.07.002

[41] Lal A. and Singh B.N. (2009): *Stochastic free vibration of laminated composite plates in thermal environments*.– J. Thermoplast. Compos. Mater., vol.23, pp.57-77, doi. 10.1177/0892705708103399.

[42] Chen C.-S., Chen C.-W., Chen W.-R. and Chang Y.-C. (2013): *Thermally induced vibration and stability of laminated composite plates with temperature-dependent properties*.– Meccanica, vol.48, No.9, pp.2311-2323, doi. 10.1007/s11012-013-9750-7.

[43] Garg A. and Chalak H.D. (2019): *A review on analysis of laminated composite and sandwich structures under hygrothermal conditions*.– Thin-Walled Structures, vol.142, pp.205-226, doi. 10.1016/j.tws.2019.05.005.

[44] Reddy J. N. (2003): *Mechanics of Laminated Composite Plates and Shells: Theory and Analysis, Second Edition*.– Taylor & Francis.

[45] Ounis H. (2015): *Numerical study using FEM on the effect of temperature on vibration and buckling of laminated composite plates*.– Ph.D., Université Mohamed Khider-Biskra, Biskra, Algérie.

[46] Sakami S. (2008): *Numerical modeling of multilayer composite structures using a discrete Mindlin approach. The DDM (Displacement Discrete Mindlin) model*.– PhD Thesis, University of Reims Champagne Ardenne).

[47] Katili I. (1993): *A new discrete Kirchhoff-Mindlin element based on Mindlin-Reissner plate-theory and assumed shear strain fields - part II. An extended DKQ element for thick-plate bending analysis*.– International Journal for Numerical Methods in Engineering, vol.36, No.11, pp.1885-1908, doi. 10.1002/nme.1620361107.

[48] Ayad R. (2002): *Contribution to numerical modeling for the analysis of solids and structures, and for the shaping of non-Newtonian fluids. Application to packaging materials*.– Habilitation to direct research, Université de Reims.

[49] Ayad R., Talbi N. and Ghomari T. (2009): *Modified discrete Mindlin hypothesises for laminated composite structures*.– *Compos. Sci. Technol.*, vol.69, No.1, pp.125-128, doi. 10.1016/j.compscitech.2007.10.038.

[50] Sedira L., Ayad R., Sabhi H., Hecini M. and Sakami S. (2012): *An enhanced discrete Mindlin finite element model using a zigzag function*.– European Journal of Computational Mechanics/Revue Européenne de Mécanique Numérique, vol.21, No.1-2, pp.122-140, doi. 10.1080/17797179.2012.702434.

[51] Chen L.-W. and Chen L.-Y. (1989): *Thermal buckling behaviour of laminated composite plates with temperature-dependent properties*.– Compos. Struct., vol.13, No.4, pp.275-287, doi. 10.1016/0263-8223(89)90012-3.

[52] Shariyat M. (2007): *Thermal buckling analysis of rectangular composite plates with temperature-dependent properties based on a layerwise theory*.– Thin-Walled Structures, vol.45, No.4, pp.439-452, doi. 10.1016/j.tws.2007.03.004.

[53] Shen H.-S. (2001): *Thermal postbuckling behaviour of imperfect shear deformable laminated plates with temperature-dependent properties*.– Computer Methods in Applied Mechanics and Engineering, vol.190, No.40, pp.5377-5390, doi. 10.1016/S0045-7825(01)00172-4.

[54] Dhanaraj R. and Palaninathan (1990): *Free vibration of initially stressed composite laminates*.– Journal of Sound and Vibration, vol.142, No.3, pp.365-378, doi. 10.1016/0022-460X(90)90656-K.

[55] Murphy K.D., Virgin L.N. and Rizzi S.A. (1997): *The effect of thermal prestress on the free vibration characteristics of clamped rectangular plates: theory and experiment*.– Journal of Vibration and Acoustics, vol.119, No.2, pp.243-249, doi. 10.1115/1.2889710.

[56] Chen L.-W. and Chen L.-Y. (1991): *Thermal postbuckling behaviours of laminated composite plates with temperature-dependent properties*.– Compos. Struct., vol.19, No.3, pp.267-283, doi. 10.1016/0263-8223(91)90031-s.

[57] Adams D.S., Bowles D.E. and Herakovich C.T. (1986): *Thermally induced transverse cracking in graphite-epoxy cross-ply laminates*.– *J.* Reinf. Plast. Compos., vol.5, No.3, pp.152-169.

[58] Nagarkar A.P. and Herakovich C.T. (1979): *Nonlinear Temperature Dependent Failure Analysis of Finite Width Composite Laminates*.– DTIC Document.

[59] Hyer M.W., Herakovich C.T., Milkovich S.M. and Short Jr J.S. (1983): *Temperature dependence of mechanical and thermal expansion properties of T300/5208 graphite/epoxy*.– Composites, vol.14, No.3, pp.276-280, doi. 10.1016/0010-4361(83)90016-2.

[60] Jones R. M. (2005): *Thermal buckling of uniformly heated unidirectional and symmetric cross-ply laminated fiber-reinforced composite uniaxial in-plane restrained simply supported rectangular plates*.– Composites Part A: Applied Science and Manufacturing, vol.36, No.10, pp.1355-1367, doi. 10.1016/j.compositesa.2005.01.028.

[61] Jones R.M. (2006): *Buckling of Bars, Plates, and Shells*.– Bull Ridge Publishing.

[62] Whitney J.M. and Ashton J.E. (1971): *Effect of environment on the elastic response of layered composite plates*.– AIAA Journal, vol.9, No.9, pp.1708-1713, doi. 10.2514/3.49976.

Received: November 30, 2023
Revised: May 9, 2024