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TWO-DIMENSIONAL DEFORMATION OF AN ELASTIC LAYER SUBJECTED TO SURFACE LOADS

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The two-dimensional deformation of a uniform, elastically isotropic layer of a finite thickness (FT) over a rough-rigid base caused by surface loads has been solved analytically. The stresses and the displacements for an isotropic layer are obtained in the integral form by applying the Airy's stress function approach. Using the appropriate boundary conditions, the problem of surface loads is discussed in detail. The integrals cannot be evaluated analytically due to the complicated expression of denominator. The linear combination of exponential terms occurring in the denominator is expanded by a finite sum of exponential terms (FSET) using the method of least squares and then the integrals are evaluated analytically. The analytical displacements are obtained for normal strip loading, normal line loading and shear line loading. The displacements have been plotted numerically for normal strip loading to find the effect of layer thickness for a Poissonian layer and compared with the half-space.

Key words: Airy stress function; two-dimensional deformation; least square approximation; rough-rigid base; surface loads.

1. Introduction

The study of the response of elastic materials due to loading is vital in engineering, soil mechanics and geophysics. Several researchers have solved the deformation problem of an elastically isotropic or transversely isotropic or orthotropic medium due to surface loads including plane strain or antiplane deformation as well as axisymmetric deformation.

The deformation problem of a layered elastic material under strip loading is solved by Small and Booker [1-2] using Fourier Hankel transform approach. The transfer matrix approach has been used by Garg and Singh [3-4] for the static deformation of a stratified elastically medium caused by surface loads considering plane strain and antiplane strain case. The results have been extended by Chaudhuri and Bhowal [5] assuming the elastic parameters varying exponentially with depth. The static deformation of a transversely isotropic layered half-space under the action of general surface loads is discussed by Pan [6] using the propagator matrix method. Extending the results of [3] for the static deformation of a stratified elastically orthotropic medium caused by surface loads is obtained by Garg *et al.* [7]. The problem of inhomogeneous transversely isotropic half-space has been discussed by Wang *et al.* [8] with Young's and shear moduli varying exponentially with depth subjected to a vertical point load. The stress field for a monoclinic elastic plate resting over different bases due to strip-loading has been obtained by Madan *et al.* [9].The analytical solution for the stresses in a transversely isotropic elastic half-plane surface subjected to loading in any direction is discussed by Ji *et al.* [10]. Recently,the two-dimensional wave propagation in a periodic plate has been studied using finite element method by Pany [11].

In the present paper, the plane strain deformation of a uniform, elastically isotropic layer of FT lying over a rough-rigid base subjected to surface loadings. The integral expressions for the deformation field have been obtained due to surface loadings by applying the Airy stress function approach. Using the appropriate boundary conditions, the stresses and displacements in the integral form are obtained for normal strip loading,

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normal line loading and shear line loading. The linear combination of exponential terms occurring in the denominator term has been approximated by a FSET [12] by applying the method of least squares [13] to solve the integrals analytically. The displacement field has been obtained for normal strip loading, normal line loading and shear line loading. The results coincide with the corresponding results for a half-space, when the thickness of the layer tends to infinity. Graphs showing the variation of displacement field have been plotted for normal strip loading to find the effect of layer thickness for Poissonian layer and compared with the half-space.

The deformation of an elastic layer/plate due to surface loads have been solved by [9,14-15] for antiplane strain case. The deformation filed is obtained analytically by [3] for a uniform half-space due to shear line loading and normal line loading for plane strain case while, in this paper ,the deformation field is obtained analytically for an elastically isotropic layer for plane strain case. The results can be extended for an anisotropic case. The problem may find applications in geophysical engineering or soil mechanics. The deformation subjected to loading such as normal line load or normal strip-load or shear line load, is useful in analyzing the deformation field due to mining tremors and drilling into the crust of the Earth. It may be significant in the prediction of seismic hazards caused by reservoir loadings or seamount loadings or glaciers or foundation footings.

2. Mathematical formulation

In the Cartesian coordinate system (x, y, z), consider a two-dimensional approximation in which the displacement components (u_x, u_y, u_z) are independent of a Cartesian coordinate x so that $\partial/\partial x \equiv 0$. Under this supposition, the plane strain problem $u_x = 0$ and the antiplane strain case ($u_y = u_z = 0$) get decoupled and therefore, can be treated independently. Here, the plane strain case is considered only.Suppose a homogeneous elastically isotropic layer of finite thickness H occupying the region $-\infty < y < \infty, 0 \le z \le H$ with z-axis vertically downwards. The upper surface $z = 0$ of the layer is acted upon by surface loads and the base $z = H$ rests upon a rough rigid surface (Fig.1(a)).

Let the elastic constants, shear modulus (μ) and the Poisson's ratio (ν) be chosen to characterize the elastically isotropic medium. The constitutive equations for an elastic isotropic medium are [16]:

$$p_{yy} = 2\mu\left(e_{yy} + \frac{\nu}{1-2\nu}\theta\right), \quad p_{zz} = 2\mu\left(e_{zz} + \frac{\nu}{1-2\nu}\theta\right), \quad p_{yz} = 2\mu e_{yz},$$
$$p_{xx} = v\left(p_{yy} + p_{zz}\right), \quad p_{xy} = p_{zx} = 0. \tag{2.1}$$

where $p_{ij}\,(i, j = x, y, z)$ are the stress components, $e_{ij}\,(i, j = x, y, z)$ are the strain components and

$$\theta = \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \tag{2.2}$$

denotes the dilatation. The plane strain problem can be formulated in terms of Airy's stress function U such that [17]:

$$p_{yy} = \frac{\partial^2 U}{\partial z^2}, \quad p_{zz} = \frac{\partial^2 U}{\partial y^2}, \quad p_{yz} = -\frac{\partial^2 U}{\partial y \partial z}, \tag{2.3}$$

satisfying

$$\nabla^2 \nabla^2 U = 0, \tag{2.4}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}. \tag{2.5}$$

The displacements in terms of Airy stress function U are given by:

$$2\mu u_y = -\frac{\partial U}{\partial y} + \frac{1}{2\alpha}\int \nabla^2 U \, dy, \tag{2.6}$$

$$2\mu u_z = -\frac{\partial U}{\partial z} + \frac{1}{2\alpha}\int \nabla^2 U \, dz, \tag{2.7}$$

where

$$\alpha = \frac{1}{2(1-2\nu)}. \tag{2.8}$$

Taking Fourier transform of biharmonic Eq.(2.4) with respect to y and solving the corresponding ordinary differential equation, a solution for the layer $-\infty < y < \infty, 0 \le z \le H$ is of the form [3]:

$$U = \int_0^\infty \left[(C_1 + C_2 kz)e^{kz} + (C_3 + C_4 kz)e^{-kz} \right] \begin{pmatrix} \sin ky \\ \cos ky \end{pmatrix} dk, \tag{2.9}$$

where C_i $(i = 1,2,3,4)$ are unknowns and may be functions of k. From Eqs (2.3),(2.6)-(2.7) and (2.9), the expressions for the stresses and the displacements are as follows:

$$p_{yy} = \int_0^\infty \left[\{C_1 + (2+kz)C_2\}e^{kz} + \{C_3 + (-2+kz)C_4\}e^{-kz} \right] \begin{pmatrix} \sin ky \\ \cos ky \end{pmatrix} k^2 dk, \tag{2.10}$$

$$p_{yz} = \int_0^\infty \left[-\{C_1 + (1+kz)C_2\}e^{kz} + \{C_3 - (1-kz)C_4\}e^{-kz} \right] \begin{pmatrix} \cos ky \\ -\sin ky \end{pmatrix} k^2 dk, \tag{2.11}$$

$$p_{zz} = -\int_0^\infty \left[(C_1 + kzC_2)e^{kz} + (C_3 + kzC_4)e^{-kz} \right] \begin{pmatrix} \sin ky \\ \cos ky \end{pmatrix} k^2 dk, \tag{2.12}$$

$$2\mu u_y = -\int_0^\infty \left[\left\{ C_1 + C_2\left(kz + \frac{1}{\alpha}\right) \right\} e^{kz} + \left\{ C_3 + C_4\left(kz - \frac{1}{\alpha}\right) \right\} e^{-kz} \right] \begin{pmatrix} \cos ky \\ -\sin ky \end{pmatrix} k\, dk, \tag{2.13}$$

$$2\mu u_z = \int_0^\infty \left[-\left\{ C_1 + C_2\left(1 - \frac{1}{\alpha} + kz\right)\right\} e^{kz} + \left\{ C_3 + C_4\left(-1 + \frac{1}{\alpha} + kz\right)\right\} e^{-kz} \right] \begin{pmatrix} \sin ky \\ \cos ky \end{pmatrix} k\,dk\,. \quad (2.14)$$

3. Surface loads

For prescribed surface loads, the boundary conditions are of the form:

$$p_{yz} = \left(p_{yz}\right)_0, \quad p_{zz} = \left(p_{zz}\right)_0 \quad (3.1)$$

at $z = 0$. Let

$$\left(p_{yz}\right)_0 = \int_0^\infty S_0 \begin{pmatrix} \cos ky \\ -\sin ky \end{pmatrix} k\,dk\,, \quad (3.2)$$

$$\left(p_{zz}\right)_0 = \int_0^\infty N_0 \begin{pmatrix} \sin ky \\ \cos ky \end{pmatrix} k\,dk. \quad (3.3)$$

Since the base $z = H$ of the elastic layer rests upon a rough- rigid base. Therefore,

$$u_y = u_z = 0 \quad (3.4)$$

at $z = H$. From Eqs (2.10)-(3.4), a system of following equations in four unknowns $C_i\,(i = 1,2,3,4)$ is obtained. Using the Cramer's rule to solve the system of equations, the following values of $C_i\,(i = 1,2,3,4)$, are obtained given below:

$$\begin{aligned}
C_1 &= \frac{1}{\delta k D}\left[S_0\left(\frac{\delta^2 - 1}{2} + 2k^2H^2\right)e^{-2kH} + N_0\left\{\left(\frac{\delta^2 + 1}{2} + 2kH + 2k^2H^2\right)e^{-2kH} - \delta e^{-4kH}\right\}\right], \\
C_2 &= \frac{1}{\delta k D}\left[S_0\left\{(1 - 2kH)e^{-2kH} - \delta e^{-4kH}\right\} - N_0\left\{(1 + 2kH)e^{-2kH} - \delta e^{-4kH}\right\}\right], \\
C_3 &= \frac{1}{\delta k D}\left[S_0\left(\frac{1 - \delta^2}{2} - 2k^2H^2\right)e^{-2kH} + N_0\left\{\left(\frac{\delta^2 + 1}{2} - 2kH + 2k^2H^2\right)e^{-2kH} - \delta\right\}\right], \\
C_4 &= \frac{1}{\delta k D}\left[S_0\left\{(1 + 2kH)e^{-2kH} - \delta\right\} + N_0\left\{(1 - 2kH)e^{-2kH} - \delta\right\}\right],
\end{aligned} \quad (3.5)$$

where

$$D = 1 + \left(A + Bk^2H^2\right)e^{-2kH} + e^{-4kH}\,, \quad A = -\left(\delta + \frac{1}{\delta}\right), \quad B = -\frac{4}{\delta}, \quad \delta = 4\nu - 3 = 1 - \frac{2}{\alpha}. \quad (3.6)$$

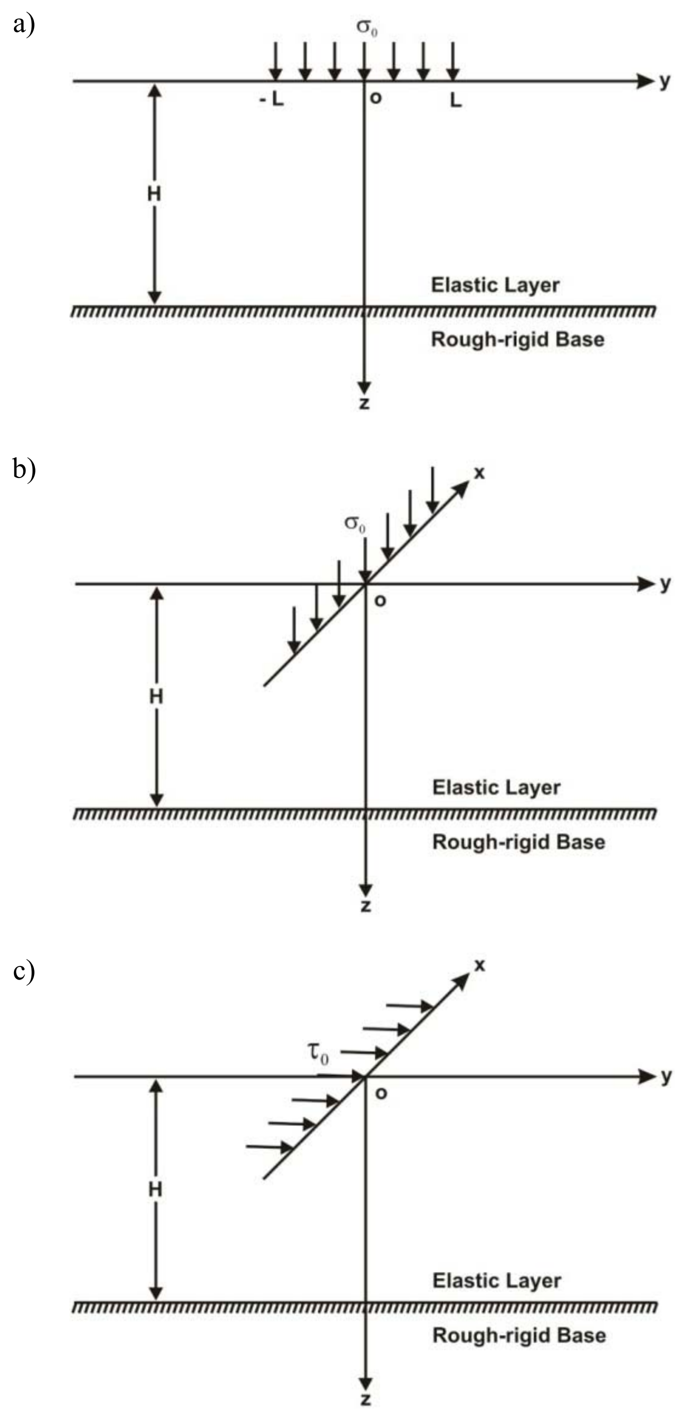

Fig.1. A uniform, elastically isotropic layer of finite thickness H overlying a rough-rigid base being acted upon by (a) normal strip loading σ_0 in the positive z-direction uniformly distributed over the strip $-L \le y \le L$ (b) normal line loading σ_0 per unit length in the positive z-direction (c) shear line loading τ_0 per unit length in the positive y-direction.

3.1. Normal strip loading

Let a normal load σ_0 per unit length be acting in the positive z-direction be uniformly distributed over the strip $-L \le y \le L$ of infinite length in the x-direction on the surface $z=0$ of the layer (Fig.1a). The boundary condition at the surface $z=0$ yields [18]:

$$p_{yz}=0, \quad p_{zz}=\frac{-\sigma_0}{2L}H\left(L-|y|\right), \tag{3.7}$$

where $H\left(L-|y|\right)$ is the Heaviside unit step function. Then

$$p_{zz}=\frac{-\sigma_0}{\pi}\int_0^{\infty}\cos ky\left(\frac{\sin kL}{kL}\right)dk\,. \tag{3.8}$$

From Eqs (3.2)-(3.3) and (3.7)-(3.8) yield

$$S_0=0, \quad N_0=\frac{-\sigma_0}{\pi k}\left(\frac{\sin kL}{kL}\right) \tag{3.9}$$

with lower solution. Substituting the values of S_0 and N_0 in Eqs (2.10)-(2.14), the deformation field is obtained in the integral form.

To compute the integrals analytically, expanding $1/D$ as a FSET [12] in such a way that the error is minimum. Expanding

$$\frac{1}{D}=\left[1+\left(A+Bk^2H^2\right)e^{-2kH}+e^{-4kH}\right]^{-1}, \tag{3.10}$$

using binomial expansion and truncate the terms up to second order

$$\frac{1}{D}\simeq 1-\left(A+Bk^2H^2\right)e^{-2kH}+\left\{C+\beta(kH)^2\right\}e^{-\gamma kH}, \tag{3.11}$$

where C is a constant independent of kH and β,γ are to be determined using the least square method for nonlinear case [13]. To find C, using the asymptotic approximation, taking the limit as $kH\to 0$, which gives

$$C=\frac{A^2+A-1}{2+A}. \tag{3.12}$$

To find the values of β,γ depending upon kH and ν in Eq.(3.11), let $\nu=0.25$ assuming the layer to be Poissonian. Applying the method of least squares, the sum of squares of deviations given by:

$$\sum_i\left(a\frac{\partial f}{\partial\beta}+b\frac{\partial f}{\partial\gamma}+R_i\right)^2$$

should be minimized, where a, b are the corrections to the approximated values of β,γ respectively and

$$R_i = \left(\frac{1}{D}\right)_{estimated} - \left(\frac{1}{D}\right)_{actual}$$

are the residuals and $f = f\left(kH, \beta, \gamma, \nu\right)$ is the right hand side of Eq.(3.11). The initial approximated values of β, γ affect the convergence a lot. The best fitted values

$$\beta = 2.049, \quad \gamma = 2.8201 \tag{3.13}$$

are obtained using the iterative process. Figure 2 shows the graph of actual curve $1/D$ given in Eq.(3.10) and the estimated curve $1/D$ given in Eq.(3.11) for best fitted values given in Eq.(3.13).

Substituting $1/D$ from Eq.(3.11), the deformation field given by Eqs (2.10)-(2.14) is represented as a linear combination of integrals, which have been solved by using standard integral tables [19]. The analytical displacements are given as follows:

$$\begin{gathered} 2\mu u_y = \frac{\sigma_0}{\pi\delta L}\left[\delta\left(\frac{\alpha-1}{2\alpha}\right)\left\{z\log\frac{R}{S} + L\tan^{-1}\left(\frac{2yz}{z^2+L^2-y^2}\right) + y\tan^{-1}\left(\frac{2Lz}{z^2-L^2+y^2}\right)\right\} + \right. \\ \left. + \frac{\delta z}{2}\log\frac{S}{R} + X_1 - AX_2 - BH^2X_3 + CX_4 + \beta H^2 X_5\right], \end{gathered} \tag{3.14}$$

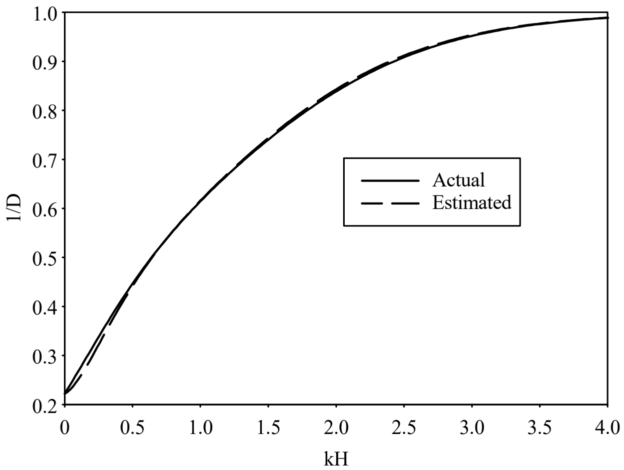

Fig.2. Variation of the actual curve and the estimated curve of $1/D$ for best fitted values $\beta = 2.049, \gamma = 2.8201$.

$$\begin{gathered} 2\mu u_z = \frac{-\sigma_0}{\pi\delta L}\left[\frac{\delta}{2\alpha}\left\{(L-y)\log R + z\tan^{-1}\left(\frac{L-y}{z}\right) + (L+y)\log S + z\tan^{-1}\left(\frac{L+y}{z}\right)\right\} - \right. \\ \left. + \frac{\delta z}{2}\tan^{-1}\left(\frac{2Lz}{z^2-L^2+y^2}\right) + X_6 - AX_7 - BH^2X_8 + CX_9 + \beta H^2 X_{10}\right], \end{gathered} \tag{3.15}$$

where

$$R^2 = z^2 + (L-y)^2, \quad S^2 = z^2 + (L+y)^2 \tag{3.16}$$

and $X_i\left(i = 1, 2, .., 10\right)$ are given in Appendix. The stresses can be computed similarly.

Taking the limit as $H \to \infty$, the displacement expressions given in Eqs (3.14)-(3.15) reduce for a uniform half space given below:

$$2\mu u_y = \frac{\sigma_0}{\pi L}\left[\left(\frac{\alpha - 1}{2\alpha}\right)\left\{z \log\frac{R}{S} + L \tan^{-1}\left(\frac{2yz}{z^2 + L^2 - y^2}\right) + y \tan^{-1}\left(\frac{2Lz}{z^2 - L^2 + y^2}\right)\right\} + \frac{z}{2}\log\frac{S}{R}\right], \tag{3.17}$$

$$\begin{aligned} 2\mu u_z = \frac{-\sigma_0}{\pi L}\Bigg[& \frac{1}{2\alpha}\left\{(L-y)\log R + z \tan^{-1}\left(\frac{L-y}{z}\right) + (L+y)\log S + z \tan^{-1}\left(\frac{L+y}{z}\right)\right\} - \\ & + \frac{z}{2}\tan^{-1}\left(\frac{2Lz}{z^2 - L^2 + y^2}\right)\Bigg] \end{aligned} \tag{3.18}$$

which coincide with the corresponding displacements of [3].

3.2. Normal line loading

Assume a normal line loading σ_0 per unit length is applied at the origin to the surface $z = 0$ in the positive z-direction (Fig.1b). Then at the surface $z = 0$

$$p_{yz} = 0, \tag{3.19}$$

$$p_{zz} = -\sigma_0\, \delta(y) = \frac{-\sigma_0}{\pi}\int_0^\infty \cos ky\, dk\,, \tag{3.20}$$

where $\delta(y)$ is the Dirac delta function. From Eqs (3.2)-(3.3) and (3.19)-(3.20) yield

$$S_0 = 0, \quad N_0 = \frac{-\sigma_0}{\pi k} \tag{3.21}$$

with lower solution. Substituting the values of S_0 and N_0 in Eqs (2.10)-(2.14), the deformation field is obtained in the integral form. Substituting $1/D$ given in Eq.(3.11) with best fitted values and solving the resulting integrals using standard integral tables [19], the analytical displacements are obtained as given below:

$$2\mu u_y = \frac{\sigma_0}{\pi\delta}\left[\delta\left(\frac{\alpha - 1}{\alpha}\right)\tan^{-1}\left(\frac{y}{z}\right) + \frac{\delta yz}{y^2 + z^2} + X_{11} - AX_{12} - BH^2 X_{13} + CX_{14} + \beta H^2 X_{15}\right], \tag{3.22}$$

$$2\mu u_z = \frac{-\sigma_0}{\pi\delta}\left[\frac{\delta}{2\alpha}\log\left(y^2 + z^2\right) - \frac{\delta z^2}{y^2 + z^2} + X_{16} - AX_{17} - BH^2 X_{18} + CX_{19} + \beta H^2 X_{20}\right], \tag{3.23}$$

where, $X_i\,(i = 11, 12, .., 20)$ are given in Appendix. The above results coincide with the displacements obtained on taking the limit as $L \to 0$ in Eqs (3.14)-(3.15) of normal strip loading. The stresses can be computed similarly.

Taking the limit as $H \to \infty$, the displacements given in Eqs (3.22)-(3.23) reduce for a uniform half space which agree with the corresponding results of [3].

3.3. Shear line loading

Let a shear line load τ_0 per unit length is applied at the origin to the surface $z=0$ in the positive y-direction (Fig.1c). Then at the surface $z=0$

$$p_{zz}=0, \tag{3.24}$$

$$p_{yz}=-\tau_0\,\delta(y)=\frac{-\tau_0}{\pi}\int_0^\infty \cos ky\,dk\,, \tag{3.25}$$

where $\delta(y)$ is the Dirac delta function. From Eqs (3.2)-(3.3) and (3.24)-(3.25) yields

$$S_0=\frac{-\tau_0}{\pi k},\quad N_0=0 \tag{3.26}$$

with upper solution. Substituting the values of S_0 and N_0 in Eqs (2.10)-(2.14), the integral expressions for the deformation field are found.

Substituting $1/D$ given in Eq.(3.11) with best fitted values and solving the resulting integrals using standard integral tables [19], the analytical displacements are obtained as follows:

$$2\mu u_y=\frac{-\tau_0}{\pi\delta}\left[\frac{\delta}{2\alpha}\log\left(y^2+z^2\right)+\frac{\delta z^2}{y^2+z^2}+Y_1-AY_2-BH^2Y_3+CY_4+\beta H^2Y_5\right], \tag{3.27}$$

$$2\mu u_z=\frac{-\tau_0}{\pi\delta}\left[\delta\left(\frac{\alpha-1}{\alpha}\right)\tan^{-1}\left(\frac{y}{z}\right)-\frac{\delta yz}{y^2+z^2}+Y_6-AY_7-BH^2Y_8+CY_9+\beta H^2Y_{10}\right], \tag{3.28}$$

where $Y_i\,(i=1,2,..,10)$ are given in Appendix. The stresses can be computed similarly.

Taking the limit as $H\to\infty$, the displacement expressions given in Eqs (3.27)-(3.28) reduce for a uniform half space which agree with the corresponding expressions of [3].

4. Numerical results

The displacement field is computed numerically for normal strip loading for the Poissonian isotropic layer. For this, the following normalized quantities are defined:

$$\frac{y}{L}=Y,\quad \frac{z}{L}=Z,\quad U_i=\frac{\mu u_i}{\sigma_0},\quad (i=y,z) \tag{4.1}$$

where Y is the normalized horizontal distance, Z is the normalized depth and $U_i\,(i=y,z)$ are the normalized displacements.

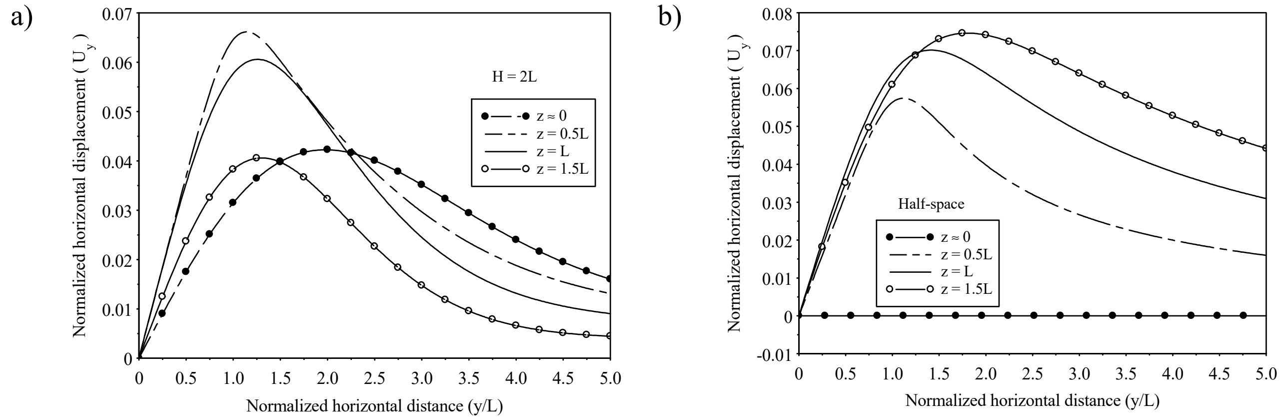

Fig.3. Graph of the normalized horizontal displacement U_y with horizontal distance y/L for different depth points for normal strip loading for (a) layer thickness $H = 2L$, (b) half-space.

Figure 3a displays the behavior of normalized horizontal displacement U_y (Eq.(3.14)) with horizontal distance y/L for different depth points $z \approx 0,\ z = 0.5L,\ L,\ 1.5L$ for a thin layer of thickness $H = 2L$. The displacement U_y has a significant behavior near $y = L$ for all depth points. Figure 3b displays the variation of U_y (Eq.(3.17)) for a uniform half-space $(H \to \infty)$ for various depth points $z \approx 0,\ z = 0.5L,\ L,\ 1.5L$. The surface displacement U_y is zero whereas the displacement U_y has a significant behavior for $z = 0.5L,\ L,\ 1.5L$.

Figure 4a shows the behavior of vertical displacement U_z (Eq.(3.15)) with the horizontal distance y/L for different points $z \approx 0,\ z = 0.5L,\ L,\ 1.5L$ for a thin layer of thickness $H = 2L$. The vertical displacement U_z initially decreases from its initial value for $z \approx 0, z = 0.5L, L$ and then increases to zero for large y/L whereas for $z = 1.5L$, the displacement increases to zero from its initial value. Figure 4b shows the variation of vertical displacement U_z (Eq.(3.18)) for a uniform half-space. The surface displacement is more dominant and decreases from its initial value rapidly as y/L increases.

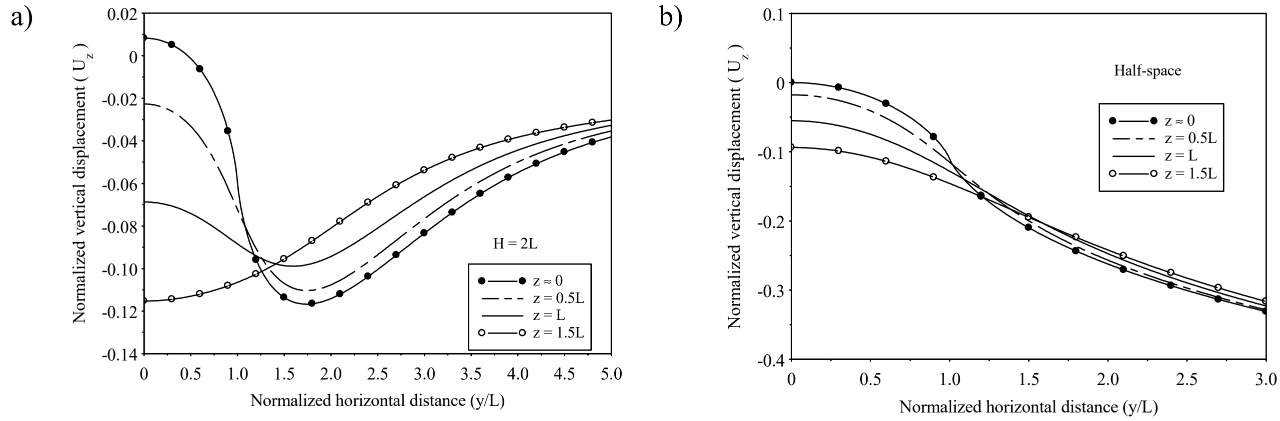

Fig.4. Graph of the normalized vertical displacement U_z with horizontal distance y/L for different depth points for normal strip loading for (a) layer thickness $H = 2L$, (b) half-space.

Figure 5a displays the contour plot of U_y in units of 10^2 with isolines drawn for layer thickness $H = 2L$. Heavy line indicates the strip loading and isovalues are also shown. Positive values of U_y are denoted by solid lines and negative values by dashed lines. The nodal lines $U_y = 0$ are also shown.

a) b)

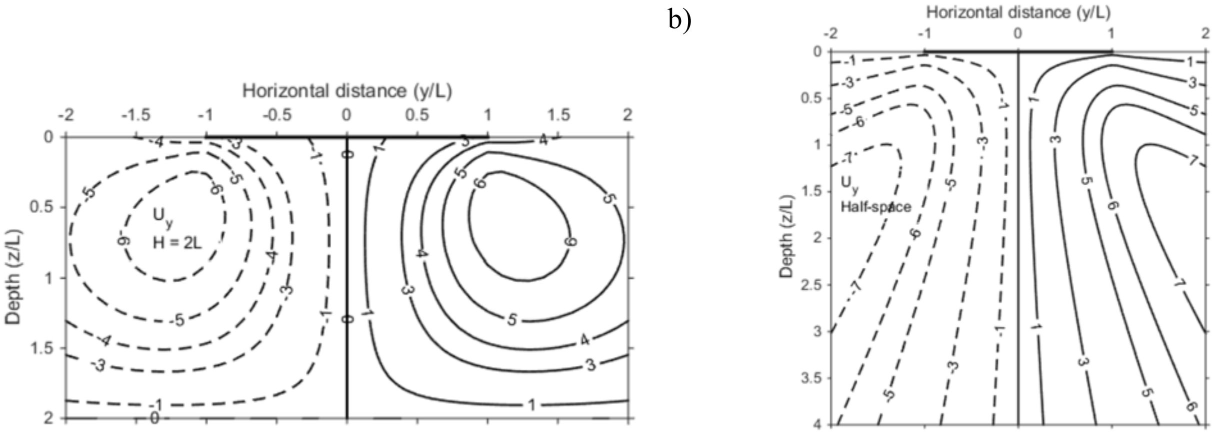

Fig.5. Contour map of the normalized horizontal displacement U_y for normal strip loading $-L \le y \le L,\ z = 0$ for (a) layer thickness $H = 2L$, (b) half-space.

a) b)

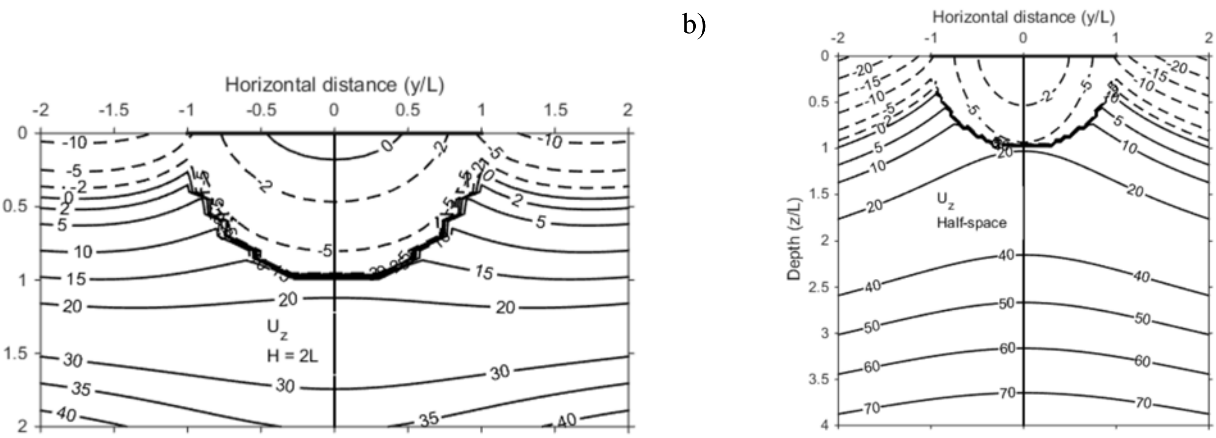

Fig.6. Contour map of the normalized vertical displacement U_z for normal strip loading $-L \le y \le L,\ z = 0$ for (a) layer thickness $H = 2L$ (b) Half-space.

Figure 5b is the contour plot of U_y for a uniform half-space. Figure 6a displays the contour plot of U_z in units of 10^2 with isolines drawn for layer thickness $H = 2L$. Heavy line indicates the strip loading and isovalues are shown. Positive values of U_z are denoted by solid lines and negative values by dashed lines. The nodal lines $U_z = 0$ are also shown. Figure 6b is the contour plot of U_z for a uniform half-space.

5. Conclusions

The plane strain deformation of a uniform, elastically isotropic layer of FT overlying a rough-rigid base subjected to normal strip loading, normal line loading and shear line loading, respectively, applied at the surface of the layer is obtained. The integrals cannot be evaluated analytically due to the complicated expression of denominator. The linear combination of exponential terms occurring in the denominator has been expanded as a FSFT using method of least squares and the integrals have been solved analytically for the displacement field. The stresses can be computed also. The results for a uniform isotropic half-space are obtained as a particular case when the thickness of the layer $H \to \infty$. Graphs showing the variation of displacement field have been plotted for normal strip loading to find the effect of layer thickness for a Poissonian layer and compared with the half-space. It is found that rough rigid base has a significant effect on the displacement field in case of a thin layer. The horizontal displacement pattern near the application of load differ significantly for a thin layer rather than a half-space. The vertical displacement (elevation) is more for a thin layer near the application of load as for the half-space.

Nomenclature

$C_i (i = 1,2,3,4)$	– unknown constants
$e_{ij} (i, j = x, y, z)$	– strain components
H	– thickness of the layer
$H(L-\lvert y\rvert)$	– Heaviside unit step function
$[-L, L]$	– strip length
$p_{ij} (i, j = x, y, z)$	– stress component
$(p_{yz})_0, (p_{zz})_0$	– surface loads
U	– Airy's stress function
$u_i (i = x, y, z)$	– displacements components
β, γ	– approximated values
$\delta(y)$	– Dirac delta function
θ	– dilatation
μ	– shear modulus
ν	– Poisson's ratio
σ_0	– normal load per unit length
τ_0	– shear load per unit length

Appendix

$$X_1 = \left| \left(\frac{1}{2\alpha} - \frac{\delta^2 + 1}{4} \right) \left\{ p_n \log \frac{S_n}{T_n} + L \tan^{-1} \left(\frac{2yp_n}{p_n^2 + L^2 - y^2} \right) + y \tan^{-1} \left(\frac{2Lp_n}{p_n^2 - L^2 + y^2} \right) \right\} - \left(H - \frac{H}{\alpha} - \frac{z}{2} \right) \log \frac{T_n}{S_n} + \right.$$
$$- \frac{4H(H-z)Lyp_n}{S_n^2 T_n^2} + \left(\frac{1}{2\alpha} - \frac{\delta^2 + 1}{4} \right) \left\{ q_n \log \frac{U_n}{V_n} + L \tan^{-1} \left(\frac{2yq_n}{q_n^2 + L^2 - y^2} \right) + y \tan^{-1} \left(\frac{2Lq_n}{q_n^2 - L^2 + y^2} \right) \right\} +$$
$$\left. + \left(H - \frac{H}{\alpha} - \frac{z}{2} \right) \log \frac{V_n}{U_n} - \frac{4H(H-z)Lyq_n}{U_n^2 V_n^2} \right|_{n=2} + \left| \delta \left(\frac{\alpha - 1}{2\alpha} \right) \left\{ p_n \log \frac{S_n}{T_n} + L \tan^{-1} \left(\frac{2yp_n}{p_n^2 + L^2 - y^2} \right) + \right. \right.$$
$$\left. \left. + y \tan^{-1} \left(\frac{2Lp_n}{p_n^2 - L^2 + y^2} \right) \right\} - \frac{\delta z}{2} \log \frac{T_n}{S_n} \right|_{n=4},$$

$$X_{12} = \left| \delta \left(\frac{\alpha - 1}{\alpha} \right) \tan^{-1} \left(\frac{y}{q_n} \right) + \frac{\delta yz}{Q_n^2} \right|_{n=2} + \left| \left(\frac{1}{\alpha} - \frac{\delta^2 + 1}{2} \right) \tan^{-1} \left(\frac{y}{p_n} \right) - \left(2H - z - \frac{2H}{\alpha} \right) \left(\frac{y}{P_n^2} \right) + \right.$$
$$\left. - \frac{4H(H-z)yp_n}{P_n^4} + \left(\frac{1}{\alpha} - \frac{\delta^2 + 1}{2} \right) \tan^{-1} \left(\frac{y}{q_n} \right) + \left(2H - z - \frac{2H}{\alpha} \right) \left(\frac{y}{Q_n^2} \right) - \frac{4H(H-z)yq_n}{Q_n^4} \right|_{n=4} +$$
$$+ \left| \delta \left(\frac{\alpha - 1}{\alpha} \right) \tan^{-1} \left(\frac{y}{p_n} \right) - \frac{\delta yz}{P_n^2} \right|_{n=6},$$

$$X_2 = \Bigg|\delta\left(\frac{\alpha-1}{2\alpha}\right)\Bigg\{q_n \log\frac{U_n}{V_n} + L\tan^{-1}\left(\frac{2yq_n}{q_n^2+L^2-y^2}\right) + y\tan^{-1}\left(\frac{2q_nL}{q_n^2-L^2+y^2}\right)\Bigg\} + \frac{\delta z}{2}\log\frac{V_n}{U_n}\Bigg|_{n=2} +$$

$$+\Bigg|\left(\frac{1}{2\alpha}-\frac{\delta^2+1}{4}\right)\Bigg\{p_n \log\frac{S_n}{T_n} + L\tan^{-1}\left(\frac{2yp_n}{p_n^2+L^2-y^2}\right) + y\tan^{-1}\left(\frac{2Lp_n}{p_n^2-L^2+y^2}\right)\Bigg\} - \left(H-\frac{H}{\alpha}-\frac{z}{2}\right)\log\frac{T_n}{S_n} +$$

$$-\frac{4H(H-z)Lyp_n}{S_n^2T_n^2} + \left(\frac{1}{2\alpha}-\frac{\delta^2+1}{4}\right)\Bigg\{q_n \log\frac{U_n}{V_n} + L\tan^{-1}\left(\frac{2yq_n}{q_n^2+L^2-y^2}\right) + y\tan^{-1}\left(\frac{2Lq_n}{q_n^2-L^2+y^2}\right)\Bigg\} +$$

$$+\left(H-\frac{H}{\alpha}-\frac{z}{2}\right)\log\frac{V_n}{U_n} - \frac{4H(H-z)Lyq_n}{U_n^2V_n^2}\Bigg|_{n=4} + \Bigg|\delta\left(\frac{\alpha-1}{2\alpha}\right)\Bigg\{p_n \log\frac{S_n}{T_n} + L\tan^{-1}\left(\frac{2yp_n}{p_n^2+L^2-y^2}\right) +$$

$$+y\tan^{-1}\left(\frac{2Lp_n}{p_n^2-L^2+y^2}\right)\Bigg\} - \frac{\delta z}{2}\log\frac{T_n}{S_n}\Bigg|_{n=6},$$

$$X_3 = \Bigg|\left(\frac{\alpha-1}{\alpha}\right)\frac{2\delta Lyq_n}{U_n^2V_n^2} + \frac{\delta z}{2}\Bigg\{2q_n^2\left(\frac{1}{U_n^4}-\frac{1}{V_n^4}\right) - \frac{1}{U_n^2} + \frac{1}{V_n^2}\Bigg\}\Bigg|_{n=2} + \Bigg|\left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\frac{2Lyp_n}{S_n^2T_n^2} +$$

$$-\left(H-\frac{H}{\alpha}-\frac{z}{2}\right)\Bigg\{2p_n^2\left(\frac{1}{S_n^4}-\frac{1}{T_n^4}\right) - \frac{1}{S_n^2} + \frac{1}{T_n^2}\Bigg\} - 2H(H-z)p_n\Bigg\{4p_n^2\left(\frac{1}{S_n^6}-\frac{1}{T_n^6}\right) - \frac{3}{S_n^4} + \frac{3}{T_n^4}\Bigg\} +$$

$$+\left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\frac{2Lyq_n}{U_n^2V_n^2} + \left(H-\frac{H}{\alpha}-\frac{z}{2}\right)\Bigg\{2q_n^2\left(\frac{1}{U_n^4}-\frac{1}{V_n^4}\right) - \frac{1}{U_n^2} + \frac{1}{V_n^2}\Bigg\} - 2H(H-z)q_n \times$$

$$\times\Bigg\{4q_n^2\left(\frac{1}{U_n^6}-\frac{1}{V_n^6}\right) - \frac{3}{U_n^4} + \frac{3}{V_n^4}\Bigg\}\Bigg|_{n=4} + \Bigg|\left(\frac{\alpha-1}{\alpha}\right)\frac{2\delta Lyp_n}{S_n^2T_n^2} - \frac{\delta z}{2}\Bigg\{2p_n^2\left(\frac{1}{S_n^4}-\frac{1}{T_n^4}\right) - \frac{1}{S_n^2} + \frac{1}{T_n^2}\Bigg\}\Bigg|_{n=6},$$

$$X_4 = \Bigg|\delta\left(\frac{\alpha-1}{2\alpha}\right)\Bigg\{q_n \log\frac{U_n}{V_n} + L\tan^{-1}\left(\frac{2yq_n}{q_n^2+L^2-y^2}\right) + y\tan^{-1}\left(\frac{2q_nL}{q_n^2-L^2+y^2}\right)\Bigg\} + \frac{\delta z}{2}\log\frac{V_n}{U_n}\Bigg|_{n=\gamma} +$$

$$+\Bigg|\left(\frac{1}{2\alpha}-\frac{\delta^2+1}{4}\right)\Bigg\{p_n \log\frac{S_n}{T_n} + L\tan^{-1}\left(\frac{2yp_n}{p_n^2+L^2-y^2}\right) + y\tan^{-1}\left(\frac{2Lp_n}{p_n^2-L^2+y^2}\right)\Bigg\} - \left(H-\frac{H}{\alpha}-\frac{z}{2}\right)\log\frac{T_n}{S_n} +$$

$$-\frac{4H(H-z)Lyp_n}{S_n^2T_n^2} + \left(\frac{1}{2\alpha}-\frac{\delta^2+1}{4}\right)\Bigg\{q_n \log\frac{U_n}{V_n} + L\tan^{-1}\left(\frac{2yq_n}{q_n^2+L^2-y^2}\right) + y\tan^{-1}\left(\frac{2Lq_n}{q_n^2-L^2+y^2}\right)\Bigg\} +$$

$$+\left(H-\frac{H}{\alpha}-\frac{z}{2}\right)\log\frac{V_n}{U_n} - \frac{4H(H-z)Lyq_n}{U_n^2V_n^2}\Bigg|_{n=2+\gamma} + \Bigg|\delta\left(\frac{\alpha-1}{2\alpha}\right)\Bigg\{p_n \log\frac{S_n}{T_n} + L\tan^{-1}\left(\frac{2yp_n}{p_n^2+L^2-y^2}\right) +$$

$$+y\tan^{-1}\left(\frac{2Lp_n}{p_n^2-L^2+y^2}\right)\Bigg\} - \frac{\delta z}{2}\log\frac{T_n}{S_n}\Bigg|_{n=4+\gamma},$$

$$X_8 = \Bigg|\frac{-\delta L\left(q_n^2-y^2+L^2\right)}{\alpha U_n^2V_n^2} - \delta z q_n\left(\frac{L-y}{U_n^4}+\frac{L+y}{V_n^4}\right)\Bigg|_{n=2} + \Bigg|\left(\frac{1-\delta^2}{2}-\frac{1}{\alpha}\right)\frac{L\left(p_n^2-y^2+L^2\right)}{S_n^2T_n^2} +$$

$$-p_n\left(2H-z+\frac{2H}{\alpha}\right)\left(\frac{L-y}{S_n^4}+\frac{L+y}{T_n^4}\right) - 2H(H-z)\Bigg\{\frac{L-y}{S_n^4}\left(\frac{4p_n^2}{S_n^2}-1\right) + \frac{L+y}{T_n^4}\left(\frac{4p_n^2}{T_n^2}-1\right)\Bigg\} +$$

$$+\left(\frac{\delta^2-1}{2}+\frac{1}{\alpha}\right)\frac{L\left(q_n^2-y^2+L^2\right)}{U_n^2V_n^2} - q_n\left(2H-z+\frac{2H}{\alpha}\right)\left(\frac{L-y}{U_n^4}+\frac{L+y}{V_n^4}\right) + 2H(H-z)\Bigg\{\frac{L-y}{U_n^4}\left(\frac{4q_n^2}{U_n^2}-1\right) +$$

$$+\frac{L+y}{V_n^4}\left(\frac{4q_n^2}{V_n^2}-1\right)\Bigg\}\Bigg|_{n=4} + \Bigg|\frac{\delta L\left(p_n^2-y^2+L^2\right)}{\alpha S_n^2T_n^2} - \delta z p_n\left(\frac{L-y}{S_n^4}+\frac{L+y}{T_n^4}\right)\Bigg|_{n=6},$$

$$X_5 = \left|\left(\frac{\alpha-1}{\alpha}\right)\frac{2\delta Lyq_n}{U_n^2V_n^2}+\frac{\delta z}{2}\left\{2q_n^2\left(\frac{1}{U_n^4}-\frac{1}{V_n^4}\right)-\frac{1}{U_n^2}+\frac{1}{V_n^2}\right\}\right|_{n=\gamma}+\left|\left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\frac{2Lyp_n}{S_n^2T_n^2}+\right.$$

$$-\left(H-\frac{H}{\alpha}-\frac{z}{2}\right)\left\{2p_n^2\left(\frac{1}{S_n^4}-\frac{1}{T_n^4}\right)-\frac{1}{S_n^2}+\frac{1}{T_n^2}\right\}-2H(H-z)p_n\left\{4p_n^2\left(\frac{1}{S_n^6}-\frac{1}{T_n^6}\right)-\frac{3}{S_n^4}+\frac{3}{T_n^4}\right\}+$$

$$+\left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\frac{2Lyq_n}{U_n^2V_n^2}+\left(H-\frac{H}{\alpha}-\frac{z}{2}\right)\left\{2q_n^2\left(\frac{1}{U_n^4}-\frac{1}{V_n^4}\right)-\frac{1}{U_n^2}+\frac{1}{V_n^2}\right\}-2H(H-z)q_n\left\{4q_n^2\left(\frac{1}{U_n^6}-\frac{1}{V_n^6}\right)+\right.$$

$$\left.\left.-\frac{3}{U_n^4}+\frac{3}{V_n^4}\right\}\right|_{n=2+\gamma}+\left|\left(\frac{\alpha-1}{\alpha}\right)\frac{2\delta Lyp_n}{S_n^2T_n^2}-\frac{\delta z}{2}\left\{2p_n^2\left(\frac{1}{S_n^4}-\frac{1}{T_n^4}\right)-\frac{1}{S_n^2}+\frac{1}{T_n^2}\right\}\right|_{n=4+\gamma}+$$

$$\left.-\left(H-\frac{z}{2}+\frac{H}{\alpha}\right)\tan^{-1}\left(\frac{2Lq_n}{q_n^2-L^2+y^2}\right)+2H(H-z)\frac{L\left(q_n^2-y^2+L^2\right)}{U_n^2V_n^2}\right|_{n=2},$$

$$X_6 = \left|\left(\frac{1}{2\alpha}+\frac{\delta^2-1}{4}\right)\left\{(L-y)\log S_n+p_n\tan^{-1}\left(\frac{L-y}{p_n}\right)+(L+y)\log T_n+p_n\tan^{-1}\left(\frac{L+y}{p_n}\right)\right\}+\right.$$

$$-\left(H-\frac{z}{2}+\frac{H}{\alpha}\right)\tan^{-1}\left(\frac{2Lp_n}{p_n^2-L^2+y^2}\right)-2H(H-z)\frac{L\left(p_n^2-y^2+L^2\right)}{S_n^2T_n^2}+$$

$$-\left(\frac{1}{2\alpha}+\frac{\delta^2-1}{4}\right)\left\{(L-y)\log U_n+q_n\tan^{-1}\left(\frac{L-y}{q_n}\right)+(L+y)\log V_n+q_n\tan^{-1}\left(\frac{L+y}{q_n}\right)\right\}+$$

$$+\left|\left(\frac{-\delta}{2\alpha}\right)\left\{(L-y)\log S_n+p_n\tan^{-1}\left(\frac{L-y}{p_n}\right)+(L+y)\log T_n+p_n\tan^{-1}\left(\frac{L+y}{p_n}\right)\right\}-\frac{\delta z}{2}\tan^{-1}\left(\frac{2Lp_n}{p_n^2-L^2+y^2}\right)\right|_{n=4}+$$

$$\left.-\left(H-\frac{z}{2}+\frac{H}{\alpha}\right)\tan^{-1}\left(\frac{2Lq_n}{q_n^2-L^2+y^2}\right)+2H(H-z)\frac{L\left(q_n^2-y^2+L^2\right)}{U_n^2V_n^2}\right|_{n=2},$$

$$X_7 = \left|\frac{\delta}{2\alpha}\left\{(L-y)\log U_n+q_n\tan^{-1}\left(\frac{L-y}{q_n}\right)+(L+y)\log V_n+q_n\tan^{-1}\left(\frac{L+y}{q_n}\right)\right\}-\frac{\delta z}{2}\tan^{-1}\left(\frac{2Lq_n}{q_n^2-L^2+y^2}\right)\right|_{n=2}+$$

$$+\left|\left(\frac{1}{2\alpha}+\frac{\delta^2-1}{4}\right)\left\{(L-y)\log S_n+p_n\tan^{-1}\left(\frac{L-y}{p_n}\right)+(L+y)\log T_n+p_n\tan^{-1}\left(\frac{L+y}{p_n}\right)\right\}+\right.$$

$$-\left(H-\frac{z}{2}+\frac{H}{\alpha}\right)\tan^{-1}\left(\frac{2Lp_n}{p_n^2-L^2+y^2}\right)-2H(H-z)\frac{L\left(p_n^2-y^2+L^2\right)}{S_n^2T_n^2}+$$

$$-\left(\frac{1}{2\alpha}+\frac{\delta^2-1}{4}\right)\left\{(L-y)\log U_n+q_n\tan^{-1}\left(\frac{L-y}{q_n}\right)+(L+y)\log V_n+q_n\tan^{-1}\left(\frac{L+y}{q_n}\right)\right\}+$$

$$\left.-\left(H-\frac{z}{2}+\frac{H}{\alpha}\right)\tan^{-1}\left(\frac{2Lq_n}{q_n^2-L^2+y^2}\right)+2H(H-z)\frac{L\left(q_n^2-y^2+L^2\right)}{U_n^2V_n^2}\right|_{n=4}+$$

$$+\left|\left(\frac{-\delta}{2\alpha}\right)\left\{(L-y)\log S_n+p_n\tan^{-1}\left(\frac{L-y}{p_n}\right)+(L+y)\log T_n+p_n\tan^{-1}\left(\frac{L+y}{p_n}\right)\right\}-\frac{\delta z}{2}\tan^{-1}\left(\frac{2Lp_n}{p_n^2-L^2+y^2}\right)\right|_{n=6},$$

$$X_{11} = \left|\left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\tan^{-1}\left(\frac{y}{p_n}\right)-\left(2H-z-\frac{2H}{\alpha}\right)\left(\frac{y}{P_n^2}\right)-\frac{4H(H-z)yp_n}{P_n^4}+\left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\tan^{-1}\left(\frac{y}{q_n}\right)+\right.$$

$$\left.+\left(2H-z-\frac{2H}{\alpha}\right)\left(\frac{y}{Q_n^2}\right)-\frac{4H(H-z)yq_n}{Q_n^4}\right|_{n=2}+\left|\delta\left(\frac{\alpha-1}{\alpha}\right)\tan^{-1}\left(\frac{y}{p_n}\right)-\frac{\delta yz}{P_n^2}\right|_{n=4},$$

$$X_9 = \left| \frac{\delta}{2\alpha}\left\{(L-y)\log U_n + q_n \tan^{-1}\left(\frac{L-y}{q_n}\right) + (L+y)\log V_n + q_n \tan^{-1}\left(\frac{L+y}{q_n}\right)\right\} - \frac{\delta z}{2}\tan^{-1}\left(\frac{2Lq_n}{q_n^2 - L^2 + y^2}\right)\right|_{n=\gamma} +$$

$$+\left|\left(\frac{1}{2\alpha}+\frac{\delta^2-1}{4}\right)\left\{(L-y)\log S_n + p_n\tan^{-1}\left(\frac{L-y}{p_n}\right) + (L+y)\log T_n + p_n \tan^{-1}\left(\frac{L+y}{p_n}\right)\right\}+\right.$$

$$-\left(H-\frac{z}{2}+\frac{H}{\alpha}\right)\tan^{-1}\left(\frac{2Lp_n}{p_n^2-L^2+y^2}\right) - 2H(H-z)\frac{L\left(p_n^2-y^2+L^2\right)}{S_n^2T_n^2}+$$

$$-\left(\frac{1}{2\alpha}+\frac{\delta^2-1}{4}\right)\left\{(L-y)\log U_n + q_n\tan^{-1}\left(\frac{L-y}{q_n}\right) + (L+y)\log V_n + q_n \tan^{-1}\left(\frac{L+y}{q_n}\right)\right\}+$$

$$\left.-\left(H-\frac{z}{2}+\frac{H}{\alpha}\right)\tan^{-1}\left(\frac{2Lq_n}{q_n^2-L^2+y^2}\right) + 2H(H-z)\frac{L\left(q_n^2-y^2+L^2\right)}{U_n^2V_n^2}\right|_{n=2+\gamma} +$$

$$+\left|\left(\frac{-\delta}{2\alpha}\right)\left\{(L-y)\log S_n + p_n\tan^{-1}\left(\frac{L-y}{p_n}\right) + (L+y)\log T_n + p_n \tan^{-1}\left(\frac{L+y}{p_n}\right)\right\} - \frac{\delta z}{2}\tan^{-1}\left(\frac{2Lp_n}{p_n^2-L^2+y^2}\right)\right|_{n=4+\gamma},$$

$$X_{10} = \left|\frac{-\delta L\left(q_n^2-y^2+L^2\right)}{\alpha U_n^2V_n^2} - \delta z q_n\left(\frac{L-y}{U_n^4}+\frac{L+y}{V_n^4}\right)\right|_{n=\gamma} + \left|\left(\frac{1-\delta^2}{2}-\frac{1}{\alpha}\right)\frac{L\left(p_n^2-y^2+L^2\right)}{S_n^2T_n^2}+\right.$$

$$-p_n\left(2H-z+\frac{2H}{\alpha}\right)\left(\frac{L-y}{S_n^4}+\frac{L+y}{T_n^4}\right) - 2H(H-z)\left\{\frac{L-y}{S_n^4}\left(\frac{4p_n^2}{S_n^2}-1\right)+\frac{L+y}{T_n^4}\left(\frac{4p_n^2}{T_n^2}-1\right)\right\}+$$

$$+\left(\frac{\delta^2-1}{2}+\frac{1}{\alpha}\right)\frac{L\left(q_n^2-y^2+L^2\right)}{U_n^2V_n^2} - q_n\left(2H-z+\frac{2H}{\alpha}\right)\left(\frac{L-y}{U_n^4}+\frac{L+y}{V_n^4}\right) + 2H(H-z)\left\{\frac{L-y}{U_n^4}\left(\frac{4q_n^2}{U_n^2}-1\right)+\right.$$

$$\left.\left.+\frac{L+y}{V_n^4}\left(\frac{4q_n^2}{V_n^2}-1\right)\right\}\right|_{n=2+\gamma} + \left|\frac{\delta L\left(p_n^2-y^2+L^2\right)}{\alpha S_n^2T_n^2} - \delta z p_n\left(\frac{L-y}{S_n^4}+\frac{L+y}{T_n^4}\right)\right|_{n=4+\gamma},$$

$$X_{13} = \left|\left(\frac{\alpha-1}{\alpha}\right)\frac{2\delta y q_n}{Q_n^4} - \frac{2\delta yz}{Q_n^4}\left(1-\frac{4q_n^2}{Q_n^2}\right)\right|_{n=2} + \left|\left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\frac{2yp_n}{P_n^4} + \left(2H-z-\frac{2H}{\alpha}\right)\frac{2y}{P_n^4}\left(1-\frac{4p_n^2}{P_n^2}\right)+\right.$$

$$+\frac{48yp_nH(H-z)}{P_n^6}\left(1-\frac{2p_n^2}{P_n^2}\right) + \left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\frac{2yq_n}{Q_n^4} - \left(2H-z-\frac{2H}{\alpha}\right)\frac{2y}{Q_n^4}\left(1-\frac{4q_n^2}{Q_n^2}\right)+$$

$$\left.+\frac{48yq_nH(H-z)}{Q_n^6}\left(1-\frac{2q_n^2}{Q_n^2}\right)\right|_{n=4} + \left|\left(\frac{\alpha-1}{\alpha}\right)\frac{2\delta yp_n}{P_n^4} + \frac{2\delta yz}{P_n^4}\left(1-\frac{4p_n^2}{P_n^2}\right)\right|_{n=6},$$

$$X_{14} = \left|\delta\left(\frac{\alpha-1}{\alpha}\right)\tan^{-1}\left(\frac{y}{q_n}\right) + \frac{\delta yz}{Q_n^2}\right|_{n=\gamma} + \left|\left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\tan^{-1}\left(\frac{y}{p_n}\right) - \left(2H-z-\frac{2H}{\alpha}\right)\left(\frac{y}{P_n^2}\right)+\right.$$

$$\left.-\frac{4H(H-z)yp_n}{P_n^4} + \left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\tan^{-1}\left(\frac{y}{q_n}\right) + \left(2H-z-\frac{2H}{\alpha}\right)\left(\frac{y}{Q_n^2}\right) - \frac{4H(H-z)yq_n}{Q_n^4}\right|_{n=2+\gamma} +$$

$$+\left|\delta\left(\frac{\alpha-1}{\alpha}\right)\tan^{-1}\left(\frac{y}{p_n}\right) - \frac{\delta yz}{P_n^2}\right|_{n=4+\gamma},$$

$$X_{15}=\left|\left(\frac{\alpha-1}{\alpha}\right)\frac{2\delta yq_n}{Q_n^4}-\frac{2\delta yz}{Q_n^4}\left(1-\frac{4q_n^2}{Q_n^2}\right)\right|_{n=\gamma}+\left|\left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\frac{2yp_n}{P_n^4}+\left(2H-z-\frac{2H}{\alpha}\right)\frac{2y}{P_n^4}\left(1-\frac{4p_n^2}{P_n^2}\right)+\right.$$
$$+\frac{48yp_nH(H-z)}{P_n^6}\left(1-\frac{2p_n^2}{P_n^2}\right)+\left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\frac{2yq_n}{Q_n^4}-\left(2H-z-\frac{2H}{\alpha}\right)\frac{2y}{Q_n^4}\left(1-\frac{4q_n^2}{Q_n^2}\right)+$$
$$\left.+\frac{48yq_nH(H-z)}{Q_n^6}\left(1-\frac{2q_n^2}{Q_n^2}\right)\right|_{n=2+\gamma}+\left|\left(\frac{\alpha-1}{\alpha}\right)\frac{2\delta yp_n}{P_n^4}+\frac{2\delta yz}{P_n^4}\left(1-\frac{4p_n^2}{P_n^2}\right)\right|_{n=4+\gamma},$$

$$X_{16}=\left|\left(\frac{1}{\alpha}+\frac{\delta^2-1}{2}\right)\log\frac{P_n}{Q_n}-\left(2H-z+\frac{2H}{\alpha}\right)\left(\frac{p_n}{P_n^2}\right)+\frac{2H(H-z)}{P_n^2}\left(1-\frac{2p_n^2}{P_n^2}\right)+\right.$$
$$\left.-\left(2H-z+\frac{2H}{\alpha}\right)\left(\frac{q_n}{Q_n^2}\right)-\frac{2H(H-z)}{Q_n^2}\left(1-\frac{2q_n^2}{Q_n^2}\right)\right|_{n=2}+\left|\left(\frac{-\delta}{\alpha}\right)\log P_n-\frac{\delta zp_n}{P_n^2}\right|_{n=4},$$

$$X_{17}=\left|\left(\frac{\delta}{\alpha}\right)\log Q_n-\frac{\delta zq_n}{Q_n^2}\right|_{n=2}+\left|\left(\frac{1}{\alpha}+\frac{\delta^2-1}{2}\right)\log\frac{P_n}{Q_n}-\left(2H-z+\frac{2H}{\alpha}\right)\left(\frac{p_n}{P_n^2}\right)+\frac{2H(H-z)}{P_n^2}\left(1-\frac{2p_n^2}{P_n^2}\right)+\right.$$
$$\left.-\left(2H-z+\frac{2H}{\alpha}\right)\left(\frac{q_n}{Q_n^2}\right)-\frac{2H(H-z)}{Q_n^2}\left(1-\frac{2q_n^2}{Q_n^2}\right)\right|_{n=4}-\left|\left(\frac{\delta}{\alpha}\right)\log P_n+\frac{\delta zp_n}{P_n^2}\right|_{n=6},$$

$$X_{18}=\left|\frac{\delta}{\alpha Q_n^2}\left(1-\frac{2q_n^2}{Q_n^2}\right)+\frac{2\delta zq_n}{Q_n^4}\left(3-\frac{4q_n^2}{Q_n^2}\right)\right|_{n=2}+\left|\left(\frac{1-\delta^2}{2}-\frac{1}{\alpha}\right)\frac{1}{P_n^2}\left(\frac{2p_n^2}{P_n^2}-1\right)+\left(2H-z+\frac{2H}{\alpha}\right)\frac{2p_n}{P_n^2}\left(3-\frac{4p_n^2}{P_n^2}\right)+\right.$$
$$-\frac{12H(H-z)}{P_n^6}\left(1-\frac{8p_n^2}{P_n^2}+\frac{8p_n^4}{P_n^4}\right)-\left(\frac{\delta^2-1}{2}+\frac{1}{\alpha}\right)\frac{1}{Q_n^2}\left(1-\frac{2q_n^2}{Q_n^2}\right)+\left(2H-z+\frac{2H}{\alpha}\right)\frac{2q_n}{Q_n^2}\left(3-\frac{4q_n^2}{Q_n^2}\right)+$$
$$\left.+\frac{12H(H-z)}{Q_n^6}\left(1-\frac{8q_n^2}{Q_n^2}+\frac{8q_n^4}{Q_n^4}\right)\right|_{n=4}+\left|\frac{-\delta}{\alpha P_n^2}\left(1-\frac{2p_n^2}{P_n^2}\right)+\frac{2\delta zp_n}{P_n^4}\left(3-\frac{4p_n^2}{P_n^2}\right)\right|_{n=6},$$

$$X_{20}=\left|\frac{\delta}{\alpha Q_n^2}\left(1-\frac{2q_n^2}{Q_n^2}\right)+\frac{2\delta zq_n}{Q_n^4}\left(3-\frac{4q_n^2}{Q_n^2}\right)\right|_{n=\gamma}+\left|\left(\frac{1-\delta^2}{2}-\frac{1}{\alpha}\right)\frac{1}{P_n^2}\left(\frac{2p_n^2}{P_n^2}-1\right)+\left(2H-z+\frac{2H}{\alpha}\right)\frac{2p_n}{P_n^4}\left(3-\frac{4p_n^2}{P_n^2}\right)+\right.$$
$$-\frac{12H(H-z)}{P_n^6}\left(1-\frac{8p_n^2}{P_n^2}+\frac{8p_n^4}{P_n^4}\right)-\left(\frac{\delta^2-1}{2}+\frac{1}{\alpha}\right)\frac{1}{Q_n^2}\left(1-\frac{2q_n^2}{Q_n^2}\right)+\left(2H-z+\frac{2H}{\alpha}\right)\frac{2q_n}{Q_n^4}\left(3-\frac{4q_n^2}{Q_n^2}\right)+$$
$$\left.+\frac{12H(H-z)}{Q_n^6}\left(1-\frac{8q_n^2}{Q_n^2}+\frac{8q_n^4}{Q_n^4}\right)\right|_{n=2+\gamma}+\left|\frac{-\delta}{\alpha P_n^2}\left(1-\frac{2p_n^2}{P_n^2}\right)+\frac{2\delta zp_n}{P_n^4}\left(3-\frac{4p_n^2}{P_n^2}\right)\right|_{n=4+\gamma},$$

$$X_{19}=\left|\left(\frac{\delta}{\alpha}\right)\log Q_n-\frac{\delta zq_n}{Q_n^2}\right|_{n=\gamma}+\left|\left(\frac{1}{\alpha}+\frac{\delta^2-1}{2}\right)\log\frac{P_n}{Q_n}-\left(2H-z+\frac{2H}{\alpha}\right)\left(\frac{p_n}{P_n^2}\right)+\frac{2H(H-z)}{P_n^2}\left(1-\frac{2p_n^2}{P_n^2}\right)+\right.$$
$$\left.-\left(2H-z+\frac{2H}{\alpha}\right)\left(\frac{q_n}{Q_n^2}\right)+\frac{2H(H-z)}{Q_n^2}\left(1-\frac{2q_n^2}{Q_n^2}\right)\right|_{n=2+\gamma}-\left|\left(\frac{\delta}{\alpha}\right)\log P_n+\frac{\delta zp_n}{P_n^2}\right|_{n=4+\gamma},$$

$$Y_1=\left|\left(\frac{\delta^2-1}{2}+\frac{1}{\alpha}\right)\log\frac{P_n}{Q_n}-\left(z-\frac{2H}{\alpha}\right)\left(\frac{p_n}{P_n^2}\right)+\frac{2H(H-z)}{P_n^2}\left(1-\frac{2p_n^2}{P_n^2}\right)-\left(z-\frac{2H}{\alpha}\right)\left(\frac{q_n}{Q_n^2}\right)+\right.$$
$$\left.-\frac{2H(H-z)}{Q_n^2}\left(1-\frac{2q_n^2}{Q_n^2}\right)\right|_{n=2}+\left|\frac{-\delta}{\alpha}\log P_n+\frac{\delta zp_n}{P_n^2}\right|_{n=4},$$

$$Y_2 = \left|\frac{\delta}{\alpha}\log Q_n + \frac{\delta z q_n}{Q_n^2}\right|_{n=2} + \left|\left(\frac{\delta^2-1}{2}+\frac{1}{\alpha}\right)\log\frac{P_n}{Q_n} - \left(z-\frac{2H}{\alpha}\right)\left(\frac{p_n}{P_n^2}\right) + \frac{2H(H-z)}{P_n^2}\left(1-\frac{2p_n^2}{P_n^2}\right) + \right.$$
$$\left. -\left(z-\frac{2H}{\alpha}\right)\left(\frac{q_n}{Q_n^2}\right) - \frac{2H(H-z)}{Q_n^2}\left(1-\frac{2q_n^2}{Q_n^2}\right)\right|_{n=4} + \left|\frac{-\delta}{\alpha}\log P_n + \frac{\delta z p_n}{P_n^2}\right|_{n=6},$$

$$Y_3 = \left|\frac{\delta}{\alpha Q_n^2}\left(1-\frac{2q_n^2}{Q_n^2}\right) - \frac{2\delta z q_n}{Q_n^4}\left(3-\frac{4q_n^2}{Q_n^2}\right)\right|_{n=2} + \left|\left(\frac{1-\delta^2}{2}-\frac{1}{\alpha}\right)\frac{1}{P_n^2}\left(\frac{2p_n^2}{P_n^2}-1\right) + \left(z-\frac{2H}{\alpha}\right)\frac{2p_n}{P_n^4}\left(3-\frac{4p_n^2}{P_n^2}\right) + \right.$$
$$-\frac{12H(H-z)}{P_n^4}\left(1-\frac{8p_n^2}{P_n^2}+\frac{8p_n^4}{P_n^4}\right) + \left(\frac{1-\delta^2}{2}-\frac{1}{\alpha}\right)\frac{1}{Q_n^2}\left(1-\frac{2q_n^2}{Q_n^2}\right) + \left(z-\frac{2H}{\alpha}\right)\frac{2q_n}{Q_n^4}\left(3-\frac{4q_n^2}{Q_n^2}\right) +$$
$$\left. +\frac{12H(H-z)}{Q_n^4}\left(1-\frac{8q_n^2}{Q_n^2}+\frac{8q_n^4}{Q_n^4}\right)\right|_{n=4} + \left|\frac{-\delta}{\alpha P_n^2}\left(1-\frac{2p_n^2}{P_n^2}\right) - \frac{2\delta z p_n}{P_n^4}\left(3-\frac{4p_n^2}{P_n^2}\right)\right|_{n=6},$$

$$Y_4 = \left|\frac{\delta}{\alpha}\log Q_n + \frac{\delta z q_n}{Q_n^2}\right|_{n=\gamma} + \left|\left(\frac{\delta^2-1}{2}+\frac{1}{\alpha}\right)\log\frac{P_n}{Q_n} - \left(z-\frac{2H}{\alpha}\right)\frac{p_n}{P_n^2} + \frac{2H(H-z)}{P_n^2}\left(1-\frac{2p_n^2}{P_n^2}\right) + \right.$$
$$\left. -\left(z-\frac{2H}{\alpha}\right)\left(\frac{q_n}{Q_n^2}\right) - \frac{2H(H-z)}{Q_n^2}\left(1-\frac{2q_n^2}{Q_n^2}\right)\right|_{n=2+\gamma} + \left|\frac{-\delta}{\alpha}\log P_n + \frac{\delta z p_n}{P_n^2}\right|_{n=4+\gamma},$$

$$Y_5 = \left|\frac{\delta}{\alpha Q_n^2}\left(1-\frac{2q_n^2}{Q_n^2}\right) - \frac{2\delta z q_n}{Q_n^4}\left(3-\frac{4q_n^2}{Q_n^2}\right)\right|_{n=\gamma} + \left|\left(\frac{1-\delta^2}{2}-\frac{1}{\alpha}\right)\frac{1}{P_n^2}\left(\frac{2p_n^2}{P_n^2}-1\right) + \left(z-\frac{2H}{\alpha}\right)\frac{2p_n}{P_n^4}\left(3-\frac{4p_n^2}{P_n^2}\right) + \right.$$
$$-\frac{12H(H-z)}{P_n^4}\left(1-\frac{8p_n^2}{P_n^2}+\frac{8p_n^4}{P_n^4}\right) + \left(\frac{1-\delta^2}{2}-\frac{1}{\alpha}\right)\frac{1}{Q_n^2}\left(1-\frac{2q_n^2}{Q_n^2}\right) + \left(z-\frac{2H}{\alpha}\right)\frac{2q_n}{Q_n^4}\left(3-\frac{4q_n^2}{Q_n^2}\right) +$$
$$\left. +\frac{12H(H-z)}{Q_n^2}\left(1-\frac{8q_n^2}{Q_n^2}+\frac{8q_n^4}{Q_n^4}\right)\right|_{n=2+\gamma} + \left|\frac{-\delta}{\alpha P_n^2}\left(1-\frac{2p_n^2}{P_n^2}\right) - \frac{2\delta z p_n}{P_n^4}\left(3-\frac{4p_n^2}{P_n^2}\right)\right|_{n=4+\gamma},$$

$$Y_6 = \left|\left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\tan^{-1}\left(\frac{y}{p_n}\right) - \left(z-2H+\frac{2H}{\alpha}\right)\left(\frac{y}{P_n^2}\right) - \frac{4H(H-z)yp_n}{P_n^4} + \left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\tan^{-1}\left(\frac{y}{q_n}\right) + \right.$$
$$\left. +\left(z-2H+\frac{2H}{\alpha}\right)\left(\frac{y}{Q_n^2}\right) - \frac{4H(H-z)yq_n}{Q_n^4}\right|_{n=2} + \left|\delta\left(\frac{\alpha-1}{\alpha}\right)\tan^{-1}\left(\frac{y}{p_n}\right) + \frac{\delta yz}{P_n^2}\right|_{n=4},$$

$$Y_7 = \left|\delta\left(\frac{\alpha-1}{\alpha}\right)\tan^{-1}\left(\frac{y}{q_n}\right) - \frac{\delta yz}{Q_n^2}\right|_{n=2} + \left|\left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\tan^{-1}\left(\frac{y}{p_n}\right) - \left(z-2H+\frac{2H}{\alpha}\right)\left(\frac{y}{P_n^2}\right) - \frac{4H(H-z)yp_n}{P_n^4} + \right.$$
$$\left. +\left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\tan^{-1}\left(\frac{y}{q_n}\right) + \left(z-2H+\frac{2H}{\alpha}\right)\left(\frac{y}{Q_n^2}\right) - \frac{4H(H-z)yq_n}{Q_n^4}\right|_{n=4} + \left|\delta\left(\frac{\alpha-1}{\alpha}\right)\tan^{-1}\left(\frac{y}{p_n}\right) + \frac{\delta yz}{P_n^2}\right|_{n=6},$$

$$Y_8 = \left|\left(\frac{\alpha-1}{\alpha}\right)\frac{2\delta y q_n}{Q_n^4} + \frac{2\delta yz}{Q_n^4}\left(1-\frac{4q_n^2}{Q_n^2}\right)\right|_{n=2} + \left|\left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\frac{2yp_n}{P_n^4} + \left(z-2H+\frac{2H}{\alpha}\right)\frac{2y}{P_n^4}\left(1-\frac{4p_n^2}{P_n^2}\right) + \right.$$
$$+\frac{48H(H-z)yp_n}{P_n^6}\left(1-\frac{2p_n^2}{P_n^2}\right) + \left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\frac{2yq_n}{Q_n^4} - \left(z-2H+\frac{2H}{\alpha}\right)\frac{2y}{Q_n^4}\left(1-\frac{4q_n^2}{Q_n^2}\right) +$$
$$\left. +\frac{48H(H-z)yq_n}{Q_n^6}\left(1-\frac{2q_n^2}{Q_n^2}\right)\right|_{n=4} + \left|\left(\frac{\alpha-1}{\alpha}\right)\frac{2\delta y p_n}{P_n^4} - \frac{2\delta yz}{P_n^4}\left(1-\frac{4p_n^2}{P_n^2}\right)\right|_{n=6},$$

$$Y_9=\left|\delta\left(\frac{\alpha-1}{\alpha}\right)\tan^{-1}\left(\frac{y}{q_n}\right)-\frac{\delta yz}{Q_n^2}\right|_{n=\gamma}+\left|\left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\tan^{-1}\left(\frac{y}{p_n}\right)-\left(z-2H+\frac{2H}{\alpha}\right)\left(\frac{y}{P_n^2}\right)-\frac{4H(H-z)yp_n}{P_n^4}+\right.$$

$$\left.+\left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\tan^{-1}\left(\frac{y}{q_n}\right)+\left(z-2H+\frac{2H}{\alpha}\right)\left(\frac{y}{Q_n^2}\right)-\frac{4H(H-z)yq_n}{Q_n^4}\right|_{n=2+\gamma}+\left|\delta\left(\frac{\alpha-1}{\alpha}\right)\tan^{-1}\left(\frac{y}{p_n}\right)+\frac{\delta yz}{P_n^2}\right|_{n=4+\gamma},$$

$$Y_{10}=\left|\left(\frac{\alpha-1}{\alpha}\right)\frac{2\delta yq_n}{Q_n^4}+\frac{2\delta yz}{Q_n^4}\left(1-\frac{4q_n^2}{Q_n^2}\right)\right|_{n=\gamma}+\left|\left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\frac{2yp_n}{P_n^4}+\left(z-2H+\frac{2H}{\alpha}\right)\frac{2y}{P_n^4}\left(1-\frac{4p_n^2}{P_n^2}\right)+\right.$$

$$+\frac{48H(H-z)yp_n}{P_n^6}\left(1-\frac{2p_n^2}{P_n^2}\right)+\left(\frac{1}{\alpha}-\frac{\delta^2+1}{2}\right)\frac{2yq_n}{Q_n^4}-\left(z-2H+\frac{2H}{\alpha}\right)\frac{2y}{Q_n^4}\left(1-\frac{4q_n^2}{Q_n^2}\right)+$$

$$\left.+\frac{48H(H-z)yq_n}{Q_n^6}\left(1-\frac{2q_n^2}{Q_n^2}\right)\right|_{n=2+\gamma}+\left|\left(\frac{\alpha-1}{\alpha}\right)\frac{2\delta yp_n}{P_n^4}-\frac{2\delta yz}{P_n^4}\left(1-\frac{4p_n^2}{P_n^2}\right)\right|_{n=4+\gamma},$$

where

$$p_n=nH-z,\quad q_n=nH+z,\quad P_n^2=y^2+p_n^2,\quad Q_n^2=y^2+q_n^2,\quad S_n^2=(L-y)^2+p_n^2,$$

$$T_n^2=(L+y)^2+p_n^2,\quad U_n^2=(L-y)^2+q_n^2,\quad V_n^2=(L+y)^2+q_n^2,\quad |f(n)|_{n=\beta}=f(\beta).$$

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