IJAME

Int. J. of Applied Mechanics and Engineering, 2024, vol.29, No.4, pp.1-23
DOI: 10.59441/ijame/192127

AN OVERVIEW OF HEAT SINK TECHNOLOGY

Ehsan F. Abbas
Technical Engineering College/ Kirkuk, Northern Technical University, IRAQ
E-mail: ehsanfadhil@ntu.edu.iq

The heat sink dissipates heat from the hot source to the environment. Also, the heat sink can absorb heat from the hot surroundings. It comes in various designs. Electric and electronic equipment, chemical industries, refrigeration, air conditioning systems, power plants, and other thermal applications use this technology. A fin array, a metal device that uses passive and active cooling techniques to dissipate heat. The surface area of the fins can be improved to enhance the efficiency of the heat sink by increasing thermal conductivity. Transverse fins can have a variety of shapes, such as a parabolic, triangular, or rectangular profile. Rectangular profiles are the most common, especially in arrays with numerous fins. The goal of this research is to look back at how longitudinal rectangular fins are optimized for arrays with fins subject to forced and natural convection work.

Key words: heat sink design, longitudinal fin, performance of a heat sink, improving fin performance, fin optimization.

1. Introduction

Heat sinks are a kind of heat exchanger that removes excess thermal energy from devices. The heat is transferred to a surrounding fluid medium, which then removes the heat. Heat sink devices are used with high-power devices as they can't regulate their temperature alone. Dissipating heat from various devices is crucial to prevent over-concentration in them, which can lead to failure or damage. Heat sinks are crucial in cooling devices, transferring heat through radiation, convection, and thermal conduction. Thermal conduction occurs when materials with different temperature levels come into contact, transferring energy from hotter to colder materials. Heat sinks transfer heat from high-temperature materials to low-temperature fluids such as air or water. In thermodynamics, heat sinks work by absorbing excess heat and maintaining a constant temperature [1]. First, we'll provide a brief overview of the innovation behind the heat sink and its developmental stages, as follows:

Between 1900 and 1920, the use of heat sink heating became widely recognized in factories and social places frequented by the upper class, such as churches and theaters. This led to the creation of multi-column cast iron relief heat sinks with high heat dissipation, which were capable of heating larger buildings. During the 1920s and 1930s, the first revolution in heat sink technology occurred with the production of single-column steel heat sinks. This significantly increased production capacity and allowed for the creation of a large number of heat sinks to meet the growing social demand. Between the 1930s and 1950s, as living standards improved, most people abandoned the primary method of making a fire to stay warm and began pursuing a higher standard of living. This led to the popularity of multi-column cast iron and multi-column steel heat sinks.

In the 1950s and 1960s, there was a significant emphasis on recovery from the aftermath of World War II. The Industrial Revolution brought about positive outcomes, resulting in an improved standard of living. People becoming more comfortable with heating demanded energy-efficient, eco-friendly, and aesthetically pleasing solutions. Copperplate heat sinks, which were affordable, had a simple and attractive design, and produced considerable heat loss, became popular among people and became a mainstream product.

During the period from 1960 to 1980, aluminium was considered to be a suitable replacement for cast iron and steel heat sinks due to its high heat transfer coefficient. However, cast aluminium profiles were too rough and straightforward, and they could not solve the problem of corrosion caused by alkaline water quality. As a result, mainstream heat sink production returned to steel during the period between 1980 and 1990. However, people desired heat sinks that matched contemporary household styles and satisfied the requirements of humanization and individuation. At that time, most manufacturers used argon arc welding technology, plug-type welding, and production line smooth tubular heat sinks, depending on the production technology level.

Columbia-Staver, established in 1972, is now one of Europe's top heat sink manufacturers. Their range of heat sinks has grown over the years, and they have incorporated new technologies to meet the changing requirements for thermal solutions in the electronics industry. Heat sinks can be classified into two types based on manufacturing and cooling methods. Each group has five subtypes, which are explained below [2, 3].

1.1. Manufacturing method

a. The stamping method.

Electrical components are cooled by air using sheet metal made of copper or aluminium, which is shaped into suitable forms. These metals have low-density thermal properties, making them ideal for cost-effective thermal solutions in high-volume production. By using high-speed stamping equipment, money can be saved, and factory-applied taps, clips, and interface materials can speed up the assembly process.

b. Extrusion.

These materials can be used to form forms that aid in heat dissipation. It is possible to create rectangular pin-fin heat sinks by employing a cross-cutting technique. To increase output, serrated fins can be added, which will lower the rate of heat extrusion by *10%* to *20%*. The limitations of extrusion determine how flexible the designs can be. Standard extrusion can produce fins with an aspect ratio of *6* and a minimum thickness of *1.3 mm*. However, a *10*-to-*1* aspect ratio with a thickness of *0.8 mm* can be obtained by employing a customized die design.

c. Bonded fins.

Air-cooled heat sinks may not work optimally when exposed to more air due to convection. However, a solution to this problem is to attach flat fins to a grooved extrusion base plate with thermally conductive aluminium-filled epoxy. This creates a high-performance heat sink that can increase the fin height-to-gap aspect ratio to *20-40*, improving cooling capacity without adding bulk.

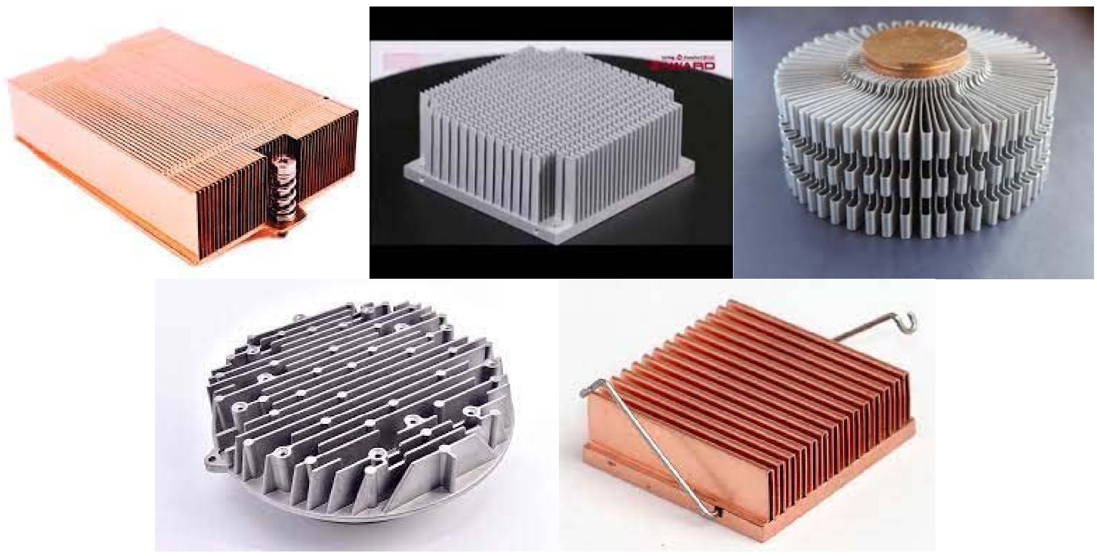


Fig.1. Illustrations of heat sink samples according to the manufacturing method.

d. Casting.

Two choices exist for casting aluminum bronze: sand casting and lost core casting, as well as die casting with or without vacuum assistance. High-density pin-fin heat sinks use impingement cooling to optimize performance.

e. Welded fins.

To improve the surface area and volumetric performance, it's possible to use corrugated copper or aluminum sheet metal. You can attach heat sinks to heating surfaces or base plates using epoxy or brazing. Keep in mind that it may not be possible to create high-profile heat sinks due to the lack of fins taller than *50 mm*, but you can still create high-performance ones through extrusion or bonding. Manufacturing companies produce heat sinks in various shapes, and some of these types are depicted in Fig.1.

1.2. Cooling method

a. Passive heat sink.

Passive heat sinks are cooling devices that operate without power and are usually bigger than active heat sinks. They utilize a larger surface area to enhance thermal cooling but require a larger area to transfer heat into the atmosphere. They frequently lessen noise, eliminate the danger of overheating from fan failure, and offer affordable solutions for larger components. However, it's important to note that they need clearance room around the installed component.

b. Active heat sink.

Active heat sinks utilize powered devices such as fans, water pumps, or air blowers to enhance cooling efficiency. They are more effective than passive heat sinks because of their reduced size and reliance on forced air circulation across the fin region. These heat sinks are commonly used in electronics to regulate temperature and are easy to install and remove.

c. Simi-active heat sink.

Leverage existing fans by placing a passive heat sink in front, producing vertical flow or impingement.

d. Liquid cooled plate heat sink.

Liquid cold plates play a vital role in a liquid cooling system. They are in charge of transferring waste heat from devices like semiconductors, microprocessors, PCBAs, or power electronics to the liquid cooling system. Compared to air cooling systems, liquid cold plates are more effective at dissipating heat due to the higher heat capacity of liquids.

e. Phase-change recirculating system.

f. The boiler and condenser in two-phase systems operate without any external power source. Heat pipe systems have wicks that work without requiring gravity feed in a gravity-fed configuration. These systems are used to control short-term temperature gradients rather than disperse heat. Cooling method heat sinks come in various shapes and sizes. Figure 2 shows some examples [4].

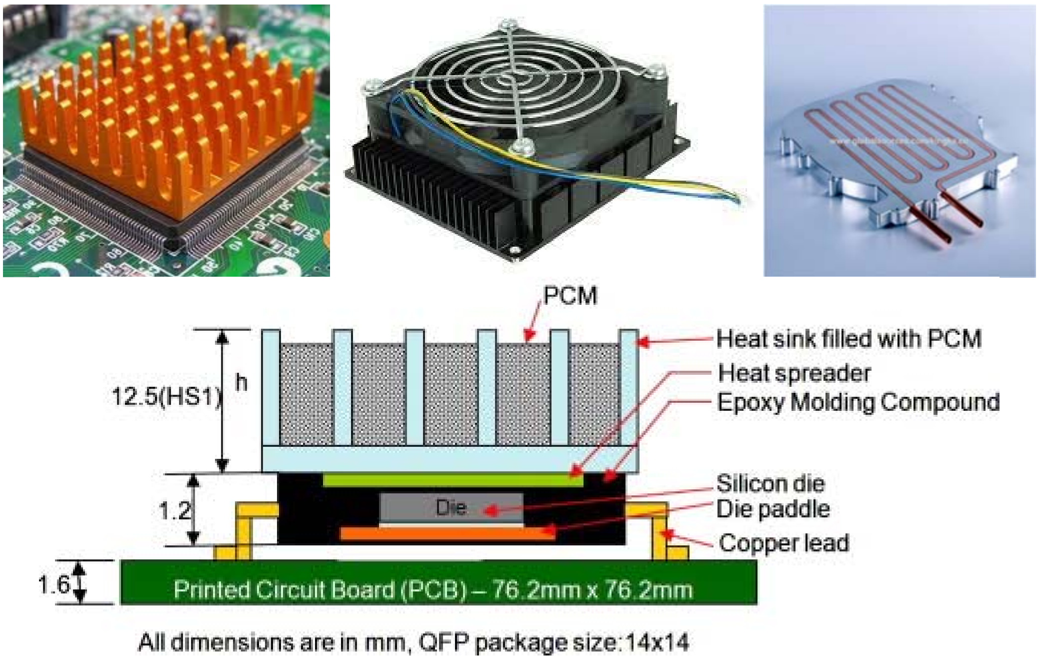


Fig.2. Illustrations of heat sink samples according to the cooling method.

2. Heat sink design

2.1. Design parameters for heat sinks

When designing or selecting a heat sink, it is crucial to consider several parameters that can impact the performance of both the heat sink and the system. The heat sink choice will depend on the thermal budget and external variables. External factors such as natural, low-flow mixed, or high-flow forced convection flow conditions play a significant role in air cooling. Certain constraints must also be taken into account when designing a heat sink, including parameters that may be limited, such as [2]:

a. Increase flow speed.
b. Specify pressure drop.
c. Select the cross-sectional area that makes sense for the flow coming in.
d. The needed heat transfers.
e. The maximum temperature at which a heat sink operates.
f. Heat sink size limitation.
g. Gravitational orientation.
h. Appearance and costs.

2.2. Assumptions made in heat sink design

The following assumptions for the heat sink are established to simplify the analysis and avoid significant calculation errors [5]:

a. The cross-sectional area of the fin may be so small that it can be considered insignificant compared to its total area.
b. The heat source and heat sink base have the same dimensions and are centrally located.
c. The heat source and the heat sink's base are in perfect contact.
d. The heat sink receives all the airflow from the fan or blower
e. Compared to convection, radiation heat transfer is negligible and can be disregarded.

f. The flow of fluid over the heat sink is laminar and consistent, without any turbulence.
g. Fins are thin compared to their spacing.
h. Constant physical properties with temperature.
i. Uniform core temperature.
j. All points with similar positions on the fin body have the same temperature.

2.3. Heat sink design fundamentals

Thermal engineers encounter the challenge of balancing performance and cost when selecting or designing a heat sink. Apart from ensuring that electronic components are at optimal temperatures, the heat sink's final design must meet size, weight, durability, and cost requirements. The basic principles of heat sinks include six fundamentals, as follows [6]:
a. Heat transfer fundamentals, thermal resistance, and preliminary computations.
b. Selecting materials for heat sinks.
c. Heat sink fin geometry.
d. Manufacturing and cost considerations for heat sinks.

3. A theoretical approach to designing heat sinks

To improve heat sink efficiency, consider increasing fin surface area, improving thermal conductivity, or enhancing the heat transfer coefficient. The shape of the fins can also play a crucial role, with parabolic, triangular, and rectangular configurations being the most commonly used. Rectangular fins are the most popular and are often used in multiple-fin arrays. A complete analysis of the optimization process for rectangular fins is available, covering both single-fin and multiple-fin arrays, as well as forced and natural convection [7][8][9].

3.1. Longitudinal rectangular profile fin

3.1.1. Temperature distribution equation

Figure 3 displays a one-dimensional longitudinal fin with a rectangular cross-sectional area. The fin is subject to the boundary conditions shown on it. An energy balance can be applied across the element with a thickness of *dx*, expressed as:

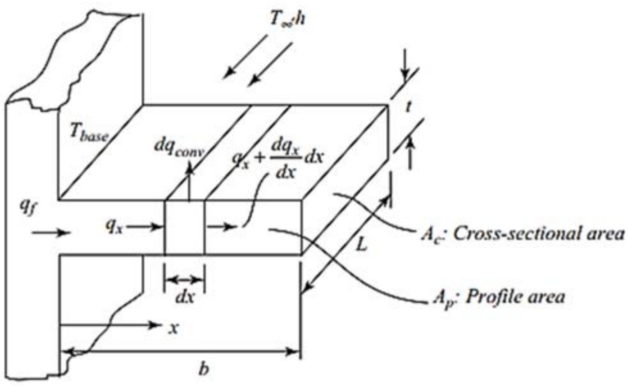


Fig.3. Longitudinal fin with a rectangular cross-section [8].

In order to calculate the energy balance for the control volume, it is imperative to account for every energy transfer that occurs within it. This balance can be expressed as:

$$q_x - \left(q_x + \frac{dq_x}{dx} dx \right) - dq_{conv} = 0 . \tag{3.1}$$

After expressing q_x using Fourier's heat conduction law and dq_{conv} using Newton's law of cooling, rearranging Eq.(3.1), it yields:

$$\frac{d^2T}{dx^2} - \frac{hp}{kA_c}\left(T - T_\infty\right) = 0. \tag{3.2}$$

Let's assume $\theta = \left(T - T_\infty\right)$ and $m^2 = \dfrac{hp}{kA_c}$, substituting them into Eq.(3.2), the result becomes:

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0. \tag{3.3}$$

The general solution for a second-order differential equation, Eq.(3.3) is as follows:

$$\theta(x) = C_1 e^{-mx} + C_2 e^{mx} . \tag{3.4a}$$

or

$$\theta(x) = C_3 \sinh mx + C_4 \cosh mx . \tag{3.4b}$$

Constants of integration C_1, C_2, C_3 and C_4 depend on the origin location, *x*-axis direction, and boundary conditions. The findings have been presented in Tab.1.

Table 1. The solution to Eqs (3.4a) and (3.4b) according to the given boundary conditions.

Case No.	Case name	Boundary conditions	Temperature distribution equation
1	Very long fin	$\theta(0) = \theta_{base}$ (a) $\theta(\infty) = 0$ (b)	$T(x) = T_\infty + \left(T_{base} + T_\infty\right) e^{mx}$ (3.5)
2	Finite length fin	$\theta(0) = \theta_{base}$ (a) $-k \left.\dfrac{d\theta}{dx}\right\rvert_{x=b} = h\theta_b$ (c)	$T(x) = T_\infty + \left(T_{base} + T_\infty\right) \times$ $\times \left[\dfrac{\cosh m(b-x) + \dfrac{h}{mk} \sinh m(b-x)}{\cosh mb + \dfrac{h}{mk} \sinh mb} \right]$ (3.6)
3	Insulated tip fin	$\theta(0) = \theta_{base}$ (a) $\left.\dfrac{d\theta}{dx}\right\rvert_{x=b} = 0$ (d)	$T(x) = T_\infty + \left(T_{base} + T_\infty\right) \dfrac{\cosh m(b-x)}{\cosh mb}$ (3.7)

3.1.2. Heat transfer from the fin

The heat dissipated from the base is transferred to the fin, which dissipates the heat into its surroundings. The temperature distribution equations can be used to calculate the heat loss from the fin to its surroundings

$$q_f = -kA_c \left.\frac{dT}{dx}\right|_{x=0} . \tag{3.8}$$

Table 2 presents the mathematical expression for heat transfer from the fin derived using Eq.(3.8) for each case in Table 1.

Table 2. Heat transfer rate formulas for each fin case.

Case name	Heat transfer rate formula
Very long fin	$q_f = \sqrt{hpkA_c}\left(T_{base} - T_\infty\right)$ (3.9)
Finite length fin	$q_f = \sqrt{hpkA_c}\left(T_{base} - T_\infty\right)\dfrac{\sinh mb + \dfrac{h}{km}\cosh mb}{\cosh mb + \dfrac{k}{mL}\sinh mb}$ (3.10)
Insulated tip fin	$q_f = \sqrt{hpkA_c}\left(T_{base} - T_\infty\right)\tanh mb$ (3.11)

3.1.3. Fin efficiency

A fin's heat transfer efficiency is determined by dividing its actual heat transfer by its maximum heat transfer. Heat transfer is maximized when the fin surface is maintained at the same temperature as the base. At this point, fins can dissipate energy at their maximum rate. It gives:

$$\eta_f = \frac{q_f}{q_{max}} . \tag{3.12}$$

The mathematical expressions for fin efficiency in different types of fin cases are:

Case 1:

$$\eta_f = \frac{1}{mb} . \tag{3.13}$$

Case 2:

$$\eta_f = \frac{1}{mb}\frac{\sinh mb + \dfrac{h}{km}\cosh mb}{\cosh mb + \dfrac{k}{mL}\sinh mb} . \tag{3.14}$$

Case 3:

$$\eta_f = \frac{1}{mb}\tanh mb . \tag{3.15}$$

A fin that is designed well can be expected to have an efficiency ranging between *0.5* and *0.7*.

3.1.4. Corrected profile length

Harper and Brown [10] found that increasing in length by half thickness represents the solution for case 2 in the same way as Eq.(3.15). Adding $t/2$ length to the fin helps place half of the fin tip area on both the top and bottom of the fin, thereby allowing convection heat transmission. This modified *Lc* is then utilized in all equations related to fins that have insulated tips. Thus

$$b_c = b + \frac{t}{2}. \tag{3.16}$$

This approximation will have an inaccuracy of less than *8%*, when:

$$\left(\frac{ht}{2k}\right)^{1/2} \leq \frac{1}{2}. \tag{3.17}$$

3.1.5. Fin effectiveness

The effectiveness of fins in enhancing heat transfer is asked. It is defined by comparing the heat transfer rate without the fin with the heat transfer rate with the fin, which can be expressed as follows:

$$\varepsilon_f = \frac{q_f}{h A_c \left(T_{base} - T_\infty\right)} \tag{3.18a}$$

or

$$\varepsilon_f = \frac{A_f}{A_c}\eta_f. \tag{3.18b}$$

The effectiveness of fins on a surface is measured by ε_f. If $\varepsilon_f = 1$, then fins have no impact on heat transfer If $\varepsilon_f < 1$, fins act as insulation. If $\varepsilon_f > 1$, fins enhance heat transfer, but they should only be used if ε_f is fins significantly larger than *1*. Finned surfaces are designed to optimize cost-effectiveness [10].

3.2. Fin optimization

The optimal characteristics of fins, such as their diameter, length, cost, and weight, have been investigated in multiple studies. These studies were carried out based on constant heat flux and fin volume. Brown [11] presented a graphical representation of the optimal dimensions for a radial fin with a rectangular profile within a practical range. Cobble [12] used a temperature distribution parameter to optimize the volume of a longitudinal fin; the minimum volume fin was found by dividing the half-fin height by the fin's volume. Snider and Kraus [13] documented the search for the most suitable longitudinal fin profile for a fin with a uniform and constant heat transfer coefficient on its surfaces. The design of longitudinal fins has been studied extensively in engineering and mathematics to achieve an optimal outcome. This problem involves determining the appropriate profile area that can maximize heat flow while using a specific weight of fin material. Schmidt [1] proposed a solution, which was later validated by Duffin [14], but was later criticized by Maday [15]. Eckert and Drake [16] made other important contributions and referenced the works of Weinig, Razelos, and Imre [17]. Previous studies have shown that the base of the fin should have a high heat flow, as b_o and t_o dissipate the most heat. Let β represent the optimal parameter, denoted as $A_p = t_o b$ which is defined as the area of the fin profile [7], [9].

$$\beta = mb_o = b_o\left(\frac{2h}{kt_o}\right)^{1/2} = A_p\left(\frac{2h}{k}\right)^{1/2}\left(\frac{1}{t_o}\right)^{3/2}. \tag{3.19}$$

The heat flow through the base of the fin can be expressed as:

$$q_f = k\,t\,L\,m\left(T_{base} - T_\infty\right)\tanh mb. \tag{3.20}$$

The unit length of the fin can be expressed using the area of the fin profile and the optimum thickness of the fin t_o.

$$q_f = kt_o\left(T_{base} - T_\infty\right)\left(\frac{2h}{kt_o}\right)^{1/2}\tanh\left[A_p\left(\frac{2h}{k}\right)^{1/2}\left(\frac{1}{t_o}\right)^{3/2}\right]. \tag{3.21}$$

By deriving Eq.(3.21) with respect to t_o and setting it to zero, one can obtain the optimal dimensions for a constant cross-sectional area, which is:

$$3\beta\,\mathrm{sech}^2\beta = \tanh\beta. \tag{3.22}$$

After solving Eq.(3.22) through trial and error, the root β is found to be *1.4192*, which can be utilized to obtain the optimum fin thickness, which expressed as:

$$t_o = \left[\frac{A_p}{1.4192}\left(\frac{2h}{k}\right)^{1/2}\right]^{2/3} \tag{3.23a}$$

or

$$t_o = 0.791\left(\frac{2hA_p^2}{k}\right)^{1/3}. \tag{3.23b}$$

The optimal relationship between fin thickness and height is determined by their profile area, so thus

$$b_o = \frac{A_p}{t_o} = 1.262\left(\frac{kA_p}{2h}\right)^{1/3}. \tag{3.24}$$

3.2.1. Constant heat flux

Aziz [18] proposed a mathematical model to optimize fin thickness and length for heat dissipation.

$$t_o = 0.632\frac{1}{hkL^2}\left[\frac{q}{\left(T_{base} - T_\infty\right)}\right]^2, \tag{3.25}$$

$$b_o = 0.7978\frac{q}{hL\left(T_{base} - T_\infty\right)}.$$

3.2.2. Constant volume

In Fig.1, the fin's mass is considered a constant which is calculated by multiplying its density by volume. The equation below can be used to calculate the fin volume, assuming that the density is constant [7]:

$$V = Lbt \tag{3.27}$$

where

$$A_p = bt = \frac{V}{L}. \tag{3.28}$$

Based on the given problem, the width L does not qualify as a parameter. Therefore, it can be considered as a constant profile area problem. Hence, the $\beta = 1.4192$ can be utilized to solve this problem. The solution can be obtained by using Eq.(3.21) along with Eq.(3.28), which is expressed by:

$$q_f = \sqrt{2hk}\, L\left(T_{base} - T_\infty\right)\sqrt{t}\ \tanh\left[\left(\frac{V}{L}\right)\sqrt{\frac{2h}{kt^3}}\right] \tag{3.29}$$

where $\beta = mb_o$.
Thus,

$$\beta = b_o\sqrt{\frac{2h}{kt_o}} = \frac{V\dfrac{b_o}{b_o t_o L}\sqrt{\dfrac{2h}{k}}}{\sqrt{t_o}}. \tag{3.30}$$

Determining the optimal fin thickness and length are possible while maintaining constant fin volume.

$$t_o \cong \left(\frac{V}{L}\right)^{\frac{2}{3}}\left(\frac{h}{k}\right)^{\frac{1}{3}}, \tag{3.31}$$

$$b_o \cong \left(\frac{V}{L}\frac{h}{k}\right)^{\frac{1}{3}}. \tag{3.32}$$

4. The thermal resistance of a multi-finned array

4.1. Thermal resistance

The fins attached to the plane wall work as a heat sink provided that there is perfect thermal contact with the wall, as shown in Fig.4. To determine the fin's resistance, we need to analyze the dissipation of heat through conduction and convection, as referenced in [8]:

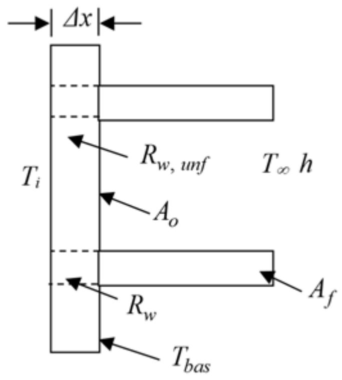


Fig.4. Sketching the heat loss from a combination of fins with a plane wall.

$$q_f = \eta_f A_f h\left(T_{base} - T_\infty\right) \tag{4.1a}$$

or

$$q_f = \frac{\left(T_{base} - T_\infty\right)}{R_f} \tag{4.1b}$$

where $R_f = \dfrac{1}{\eta_f A_f h}$.

The heat transfer rate of an unfinned area can be calculated as:

$$q_{unf} = A_{unf} h\left(T_{base} - T_\infty\right) \tag{4.2a}$$

or

$$q_{unf} = \frac{\left(T_{base} - T_\infty\right)}{R_{unf}} \tag{4.2b}$$

where $R_{unf} = \dfrac{1}{A_{unf} h}$.

To calculate the amount of heat dissipated from a heat sink, you can use the following formula:

$$q_t = q_{f,t} + q_{unf,t} \tag{4.3}$$

where

$$q_{f,t} = \frac{\left(T_i - T_\infty\right)}{n\left(R_f + R_{w,f}\right)}, \tag{4.4}$$

$$q_{unf,t} = \frac{\left(T_i - T_\infty\right)}{m\left(R_{unf} + R_{w,unf}\right)}, \tag{4.5}$$

and

$$R_{w,f} = \frac{\Delta x}{k_w A_c}, \tag{4.6}$$

$$R_{w,unf} = \frac{\Delta x}{k_w A_{unf}}, \tag{4.7}$$

$$\begin{aligned} q_t &= \left(T_i - T_\infty\right)\left[\frac{1}{n\left(R_f + R_{w,f}\right)} + \frac{1}{m\left(R_{unf} + R_{w,unf}\right)}\right] = \\ &= \left(T_i - T_\infty\right)\frac{n\left(R_f + R_{w,f}\right) + m\left(R_{unf} + R_{w,unf}\right)}{m\left(R_{unf} + R_{w,unf}\right)\times\left(R_f + R_{w,f}\right)}. \end{aligned} \tag{4.8}$$

4.2. Overall surface efficiency

The overall surface efficiency quantifies the performance of the entire array This is calculated using the efficiency of the fins in combination with the surface of the base. The overall efficiency in each case is calculated as follows [19]:

$$\eta_o = \frac{q_t}{q_{max}}. \tag{4.9}$$

Total heat transferred from the multi-fin array and plane wall can be calculated by another formula, which is expressed as:

$$q_t = \eta_f A_{f,t} h\left(T_{base} - T_\infty\right) + A_{unf,t} h\left(T_{base} - T_\infty\right), \tag{4.10}$$

$$A_{unf,t} = A_t - A_{f,t}, \tag{4.11}$$

$$q_{max} = A_t h\left(T_{base} - T_\infty\right). \tag{4.12}$$

Substituting Eqs (4.10) to (4.12) into Eq.(4.9) and rearranging it yields:

$$\eta_o = 1 - \frac{A_{f,t}}{A_t}\left(1 - \eta_f\right). \tag{4.13}$$

5. Longitudinal fin with non-unform profile area

Different longitudinal fin shapes with non-uniform profiles, like concave, convex, and triangle fins, are used in thermal equipment (Fig.5). Numerous theoretical studies have been conducted to determine the optimal dimensions for these fins. Table 3 lists research on the ideal fin size as well as formula expressions for heat transfer for earlier fin types.

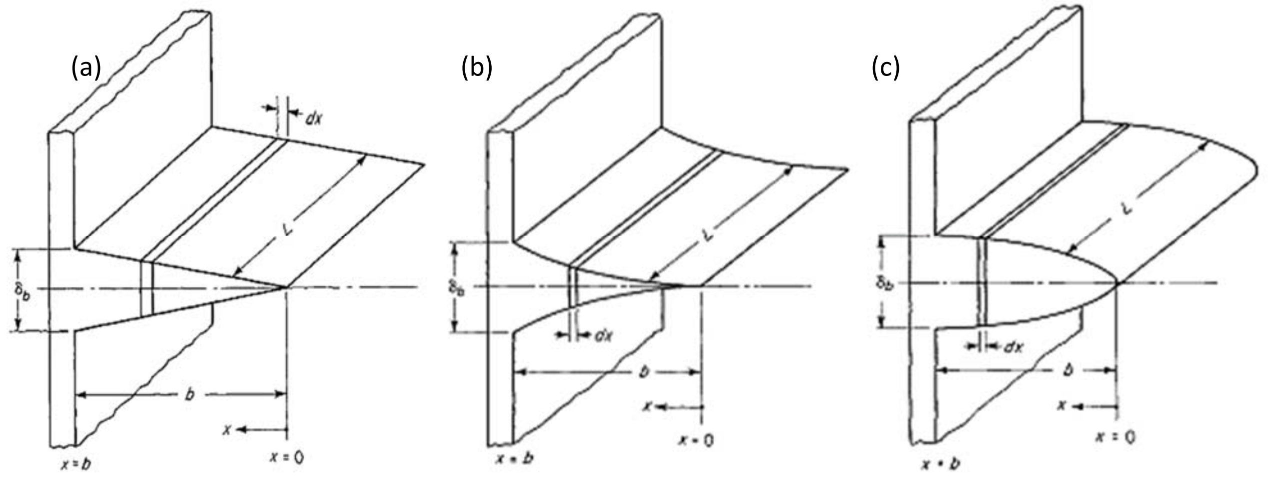


Fig.5. Sketches of longitudinal fins with non-uniform profile area: (a) triangular, (b) concave, and (c) convex [9].

Table 3. Mathematical expressions for calculating optimal dimensions and heat transfer rates of non-uniform longitudinal fins.

Fin shape	Reference	Optimum dimension equation	Heat transfer equation	A_p
Rectangular	Aziz [18]	$t_o = 0.9977\left(\frac{A_p^2 h}{k}\right)^{1/3}$ $b_o = 1.0023\left(\frac{A_p k}{h}\right)^{1/3}$	$q_f = \sqrt{2hkt}\,(T_{base} - T_\infty)\times$ $\times\tanh\left[A_p\left(\frac{2h}{k}\right)^{1/2} t^{-3/2}\right]$	tb
		$t_o = \frac{0.6321}{hk}\left(\frac{q}{T_{base} - T_\infty}\right)^2$ $b_o = 0.7978\frac{q}{h(T_{base} - T_\infty)}$		
	Kraus *et al.* [9] Chung and Iyer [20]	$t_o = 0.791\left(\frac{2A_p^2 h}{k}\right)^{1/3}$ $b_o = 1.262\left(\frac{A_p k}{2h}\right)^{1/3}$		
Triangular	Schmidt [9], [21]	$t_o = 1.673\left(\frac{A_p^2 h}{k}\right)^{1/3}$ $b_o = 1.1953\left(\frac{A_p k}{h}\right)^{1/3}$	$q_f = \sqrt{2hkt}\,(T_{base} - T_\infty)\times$ $\times\frac{I_1\left[4A_p\left(\frac{2h}{kt^3}\right)^{1/2}\right]}{I_0\left[4A_p\left(\frac{2h}{kt^3}\right)^{1/2}\right]}$	$tb/2$

Table 3 cont. Mathematical expressions for calculating optimal dimensions and heat transfer rates of non-uniform longitudinal fins.

Fin shape	Reference	Optimum dimension equation	Heat transfer equation	A_p
Triangular	Aziz [18]	$t_o = 1.671\left(\frac{A_p^2 h}{k}\right)^{1/3}$ $b_o = 1.1969\left(\frac{A_p k}{h}\right)^{1/3}$	$q_f = \sqrt{2hkt}\,(T_{base} - T_\infty)\times$ $\times\frac{I_1\left[4A_p\left(\frac{2h}{kt^3}\right)^{1/2}\right]}{I_0\left[4A_p\left(\frac{2h}{kt^3}\right)^{1/2}\right]}$	$tb/2$
		$t_o = \frac{0.8273}{hk}\left(\frac{q}{T_{base} - T_\infty}\right)^2$ $b_o = 0.8422\frac{q}{h(T_{base} - T_\infty)}$		
Concave parabolic	Aziz [18]	$t_o = 2.0801\left(\frac{A_p^2 h}{k}\right)^{1/3}$ $b_o = 1.4422\left(\frac{A_p k}{h}\right)^{1/3}$	$q_f = \frac{ktL}{2b}(T_{base} - T_\infty)\times$ $\times\left[-1 + \sqrt{\left(1 + \frac{72hA_p^2}{kt^3}\right)}\right]$	$\frac{tb}{3}$
		$t_o = \frac{\left(\frac{q}{T_{base} - T_\infty}\right)^2}{hk}$ $b_o = \frac{q}{h(T_{base} - T_\infty)}$		
	Schmidt [9]	$t_o = 2.0801\left(\frac{A_p^2 h}{k}\right)^{1/3}$ $b_o = 1.4422\left(\frac{A_p k}{h}\right)^{1/3}$		
Convex profile	Chung and Nguyen [22] Kraus *et al.* [9]	$t_o = 1.778\left(\frac{\sigma\epsilon A_p^2 T_{base}^3}{k}\right)^{1/3}$ $b_o = 0.843\left(\frac{kA_p}{\sigma\epsilon T_{base}^3}\right)^{1/3}$ $T_{\infty,o} = 0.768T_{base}$	$q_f = 0.927\times$ $\times\left[k(\sigma\epsilon)^2 A_p T_{base}^9\right]^{1/3}$	$A_p = 1.2553\times$ $\times\left[\frac{q^3}{k(\sigma\epsilon)^2 A_p T_{base}^9}\right]$

6. Array of longitudinal fins

Levy E.K. [23] conducted an optimization study to determine the optimal spacing between parallel vertical plates for natural laminar convection. This was investigated based on two studies conducted by Elenbaas [24] experimentally studied plate spacing optimization using free convection heat transfer between two parallel vertical plates based on maximum heat transfer rate, and Bodoia [25] investigated optimal plate spacing between two parallel vertical plates by solving continuity, momentum, and energy equations. This study suggests a new optimal solution to minimize the temperature differential between plates and fluid at a given heat flux. The lowest temperature difference is possible by keeping the plates far apart to prevent the wall boundary layers from merging. Elenbaas's optimum requires only *54%* of the minimum plate spacing, resulting in a temperature differential of *38%* higher than the proposed minimum. Bar-Cohen and Jelinek [22, 26] optimized a heat sink with longitudinal rectangular fins using the individually optimized fins method. As illustrated in Fig.6, the heat transfer rate from a series of longitudinal, rectangular fins can be calculated by adding the fins' thermal contribution and the unfinned area.

$$\frac{Q}{L} = n\left[\left(\frac{2hkA_p}{b}\right)^{1/2} \theta_b \tanh\beta + h_w \theta_b \left(\frac{W}{n} - \frac{A_p}{b}\right)\right] \tag{6.1}$$

where

$$\beta = mb = \sqrt{\frac{2h}{kA_p}}\, b^{\frac{3}{2}}\ .$$

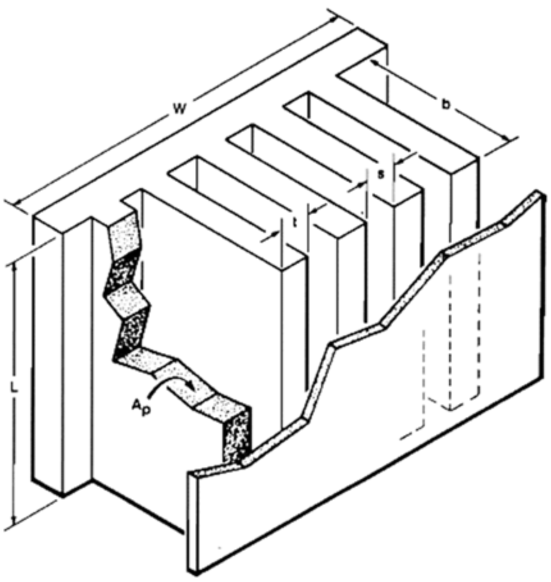


Fig.6. A sketch of an array of longitudinal fins and its geometric definitions [9], [18].

To determine the maximum amount of heat that can be dissipated by an array at a specific profile area $\left(A_p\right)$, or when the number of fins is pre-determined due to system requirements for a fixed nA_p, one needs to calculate the derivative of $\left(\frac{Q}{Ln\theta_b}\right)$ with respect to (b), starting from zero. This helps us to identify the optimal (β) value for the array.

$$\left[\sqrt{2hkA_p}\left(-\frac{1}{\sqrt{b^3}}\tanh\beta + \frac{1}{\sqrt{b}}\frac{3\beta}{2}\operatorname{sech}^2\beta\right) + \frac{h_w A_p}{b^2}\right]_{opt} = 0\,. \tag{6.2}$$

Multiply both sides of the equation by $\frac{2b^3}{kA_p}$ $2b$ and then cancel any common terms, and you get:

$$\left(\beta\tanh\beta - 3\beta^3\mathrm{sech}^2\beta\right)_{opt} = \frac{2h_w b}{k}. \tag{6.3}$$

To achieve the optimum possible outcome for (b), we can start by assuming that h_w is zero. According to evaluations made by [15] and [16], we get $\beta = 1.4192$. A linear relationship can approximate the solution to Eq.(4.10) for h_w values other than zero,

$$\beta_{opt} = 1.4192 + 1.125\frac{h_w b}{k}. \tag{6.3}$$

This equation is valid for calculating bop in a range $1.4192 < \beta_{opt} < 1.6$.

Figure 7 illustrates the relationship between optimal beta and $h_w b / k$. It can be used instead of Eq.(6.4) to obtain the optimum beta when $h_w b / k$ is known.

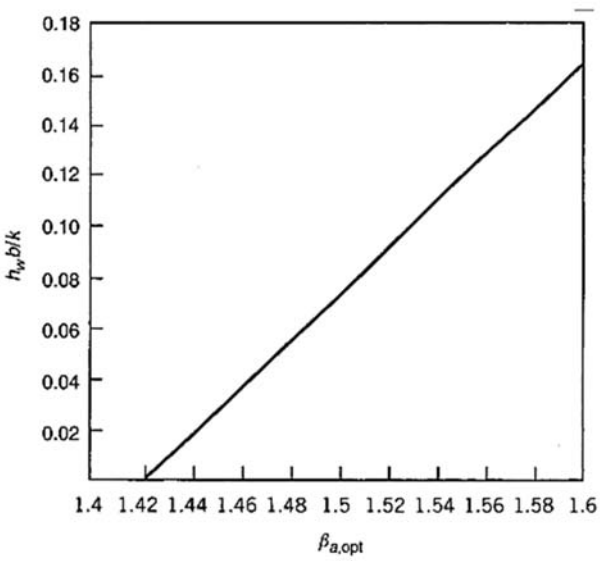


Fig.7. Optimal β values for any array of longitudinal fins [9, 18].

7. Experimental and numerical studies

Starner and McManus [27] examined the effect of a longitudinal rectangular fin array's height and spacing distance on heat sink free convection heat transfer. The study found that the end-bow conditions strongly affect the heat transfer of horizontal arrays. For the most profound channel array, preventing flow in this direction lowered heat transfer by up to *50%*. The horizontal orientation is more favorable for shallow channels due to the ample flow from above. Additional tests are necessary to obtain complete data on heat transfer from horizontal-base systems.

Jones and Smith [28] experimented on a horizontal rectangular fin heat sink to analyze free convection processes under varying fin heights and spacing distances. The study found that isothermal fins were arranged in free convection cooling arrays on flat surfaces, covering a more comprehensive range of fin spacings. The present study introduces a simplified heat transfer correlation, challenges an outdated correlation, and proposes a preliminary design method that considers weight to determine the most efficient configuration for maximizing heat transfer.

Barrett and Obinelo [29] investigated the thermal performance of a longitudinal fin heat sink using both experimental and numerical methods. They investigated how the flow bypass phenomenon influences heat sink thermal resistance and pressure drop at different approach flow rates and fin densities. They also looked at parameters like tip clearance and spanwise spacing. The study presents comprehensive findings on heat transfer and pressure drop properties.

Vollaro *et al*. [30] created a mathematical model for a rectangular fin heat sink with natural heat transfer convection. The study examined how the thermal conductivity and emissivity of materials with vertical fins affect how well a system works when heat moves naturally through convection. They discovered that decreasing fin spacing due to finite fin conductivity reduces the optimal configuration, whereas compressing the system increases heat flux by up to *20%*.

Baskaya *et al*. [31] used computational fluid dynamics (CFD) software to predict the optimal thermal performance of a longitudinal fin heat sink. The study investigated the effects of fin spacing, length, and temperature differences on heat transfer between the fin and the surrounding environment. Previous research has shown that only focusing on one or two of these factors is not enough to achieve optimal heat transfer performance. It is crucial to consider the interaction between all design parameters. The researchers compared their findings to previously collected experimental data and presented graphical representations of optimal values and correlations.

Bar-Cohen *et al*. [2] examined the number and thickness of fins for the optimum weight and type of material for a plate fin array heat sink under natural convection conditions. The optimally spaced least-material array is the most effective thermal design, suggesting that the least-material approach is a suitable optimization design heuristic. Magnesium is the most efficient material for thermal performance. However, innovative manufacturing techniques are needed to produce thermally optimal fin arrays due to the unconventional aspect ratios of these optimal arrays.

Yazicioğlu and Üncü [33] investigated the ideal vertical spacing for rectangular fin arrays in natural conditions. Fin lengths ranged from *250* to *340 mm*, fin spacing from *5.75* to *85.5 cm*, and fin heights from *5* to *25 mm*. A correlation has been obtained, which can be used to calculate the heat transfer rate.

Mittelman *et al*. [34] investigated natural convection heat transfer from an inclined longitudinal heat sink. Tilting the fin array beyond a particular angle enhances the heat transfer rate. The heat transfer coefficient is less affected by changes in fin height, but it increases with higher fin temperatures and wider fin spacing. A semi-empirical model was developed to predict the conventional heat transfer coefficient of flat or slightly tilted arrays with spaced fins.

Aziz and Abers-Green [36] conducted a study to enhance a heat sink's thermal performance by attaching rectangular fins that radiate longitudinally to a thermal wall. They used Maple software to optimize the fin's thermal performance and analyzed various factors that affect its efficiency data, including temperature distribution, heat transfer rates, and convection and radiation numbers. The study provides graphical data for fin designers to use without needing to engage in computational techniques' mathematical details.

Shaeri and Yaghoubi [35] found that using perforated fins in a heat sink improves heat transfer. As the number of perforations increases, so does the thermal performance and effectiveness of the fins. They also significantly reduce the fins' weight. The researchers proposed a new correlation to predict the effectiveness of perforated fins.

Kim K. Dong [36] studied the natural convection thermal performance of a vertical plate-fin heat sink. They optimized the heat sink for fin thickness variation perpendicular to fluid flow. Increasing fin thickness reduced thermal resistance by *10%*. The difference between uniform and variable-thickness heat sinks decreases with height or heat flux.

Ahmadi *et al*. [37] conducted a study to investigate the impact of fin interruption on natural convection. They used a 2-D numerical model created with FLUENT software and tested various aluminum alloy heatsinks with 12 different geometric dimensions to validate the theoretical results. The study included an extensive numerical and experimental examination of the interruptions and spacing of fins.

Pouryoussefi and Zhang [38] compared flat plate and circular pin fin heat sinks under forced convection. Pressure drop, thermal resistance, and overall performance were measured in the air velocity range from *4.7* to *12.5 m/s* to evaluate the heat sink's thermal and hydrodynamic performance. The study found that free-stream air velocity affects system thermal and hydrodynamic performance. The free stream's velocity increases the heat transfer coefficient, lowering thermal resistance. Fluid inertia increases pressure drop at higher velocities. The circular pin heat sink has *37.7%* lower thermal resistance than the original under the same free stream air velocity.

Karamanis and Hodes [39] developed a method to optimize the design of a longitudinal fin heat sink for better performance under laminar flow. The technique minimizes thermal resistance by optimizing fin thickness, spacing, and other parameters. It requires minimal computation and can optimize the heat sink length. A dimensionless formulation was discovered for calculating thermal resistance.

Shadlaghani *et al*. [40] found that triangular fins sink heat more efficiently than trapezoidal or rectangular fins. Increasing the height-to-thickness ratio of triangular fins enhances their heat transfer rate. Longitudinal and square or circular perforations effectively improve the thermal performance of triangular fins.

Hong and Chung [41] investigated the optimization of fin spacing variation by using numerical and experimental methods. Simulation data was validated by using mass and heat transfer analogies with *Pr* numbers between *0.7* and *2000*. It is optimal to space fins farther apart as the *Pr* number decreases. It was found that the thermal boundary layers of adjacent fins interact with one another depending on their *Pr* numbers. The heat transfer rate of a finned plate was predicted using a straightforward heat transfer correlation.

Doğan and Doğan [42] experimented with different heat sink parameters, including transfer pitch, longitudinal pitch, and fin height. The study analyzed natural convection from the fin array for different fin heights, spacing ratios $C = S_t / S_b$, and *Ra* numbers. The best ratio for maximum heat transfer was found to be between *0.50* and *0.75*, depending on fin height and *Ra*. An empirical *Nu* number correlation was derived using fin design and natural convection heat transfer.

Krishna [43] studied a heat sink with aluminum fins oriented lengthwise and a phase change material known as RT44HC. The Taguchi method optimized the heat sink configuration for the highest operational time. The best configuration was attained when the critical temperatures were set at *54.8°C*, *63°C*, and *72.6°C*

Hou *et al*. [44] studied free convection heat transfer from vertical and inclined rectangular heat sinks. They found that the convection heat transfer rate remains unchanged in the slice layout structure when the heat sink is tilted downward. However, the convection heat transfer rate significantly increases in the brick layout case due to flow separation inside the fin channels.

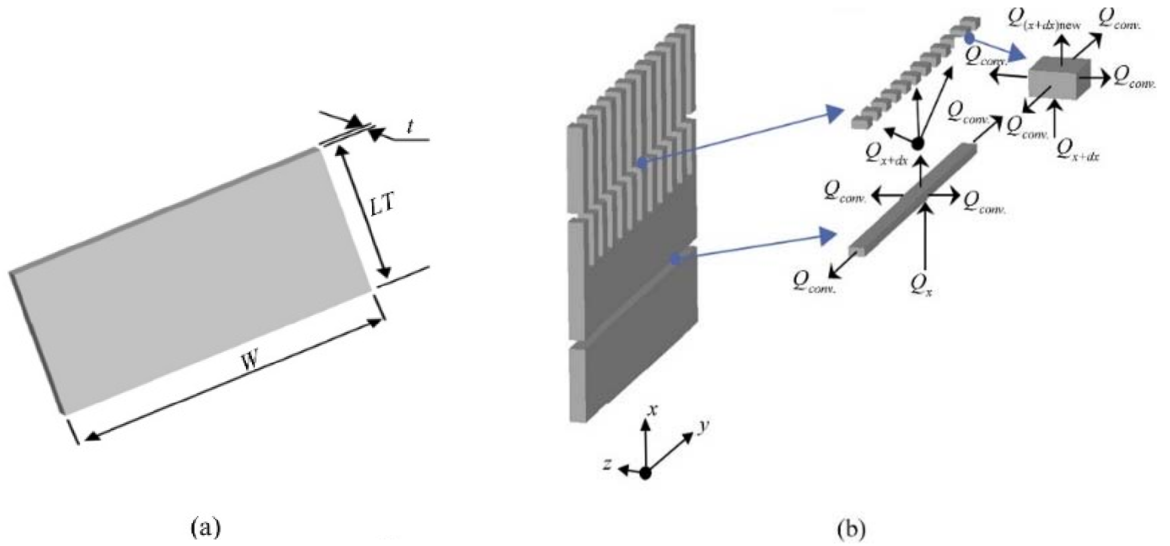
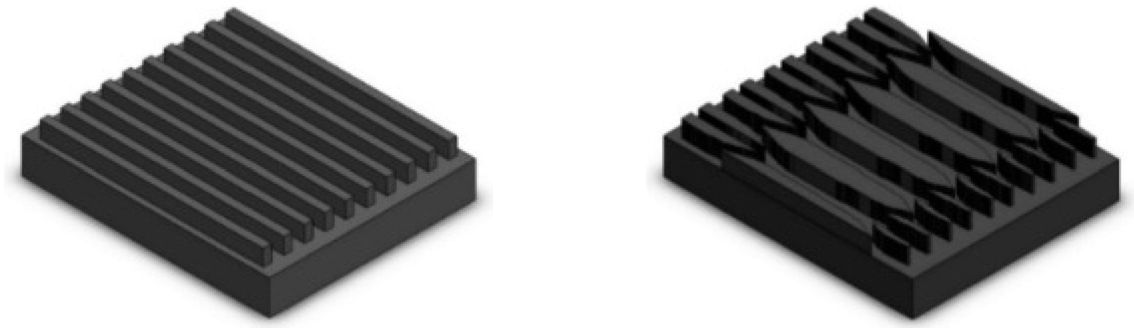


Fig.8. Sketch for two heat sink models: (a) conventional, (b) hybrid.

Jasim [45] optimized a combined fin heat sink design by merging two fin shapes vertically to enhance heat transfer. The study showed that this design reduced the temperature profile by an average of *2.7%* to *8.8%*, declined thermal resistance by *23%* to *43%*, and increased heat transfer by *29%* to *78%*. The *Nu* number improved across all operating conditions.

Xie *et al*. [46] compared a topology-optimized heat sink with a straight-fin heat sink regarding heat transfer and pressure drop performance. The optimized heat sink showed a *5.76%* decrease in average temperature, indicating potential energy savings and emissions reduction.



(a) (b)

Fig.9. Sketch of studied heat sink models, (a) straight fin, (b) topology.

Obaid and Hameed [47] investigated various fin configurations of heat sinks and compared their performance. Four types of fins were tested under three heat fluxes (*100*, *150*, and $200\ W/m^2$) at a *1.5 m/s* airflow rate: flat fins, corrugated fins, rectangular fins, and triangular fins. The temperature distribution of each studied case in experimental and numerical studies has a deviation of *2.95-3.37%* for flat and corrugated fins, and *1.41-1.2%* for rectangular and triangular fins at *160 W*. The last two types have a greater heat transfer coefficient compared to flat fins. Rectangular and triangular fins with fin perforations and ribs decrease material cost and heat sink weight by *7.8-7.3%*.

Murat and Şükrü [48] optimized the fin space for a heat sink consisting of rectangular fins. It was noted that the efficiency of heat dissipation from a heat sink was maximized when the number of fins was decreased from *100* to *66*, while the overall dimensions remained unchanged. Hence, it was deduced that the heat sink featuring *66* fins exhibited the highest heat dissipation efficiency.

8. Conclusions

This article reviews the different techniques used to optimize the design of heat sinks with rectangular longitudinal fins. Fins were created over three centuries ago to improve thermal device and equipment heat transfer. Several studies have employed experimental, numerical, and theoretical methods to determine the best heat sink configuration. Based on the findings from previous research, this paper provides a summary of the subsequent observations:

- The height and spacing of a rectangular fin affect heat transfer rates. Duralumin fin arrays perform better than stainless steel with similar geometries. Optimizing the fin spacing is essential to achieving maximum heat transfer. The heat transfer coefficient rises to a critical fin spacing value and decreases with increasing fin spacing. Reducing optimal fin spacing can change heat flux by up to *20%*.
- Inclined fins with interrupted surfaces improve heat transfer performance. Convective heat transfer increases with higher angles.
- Interrupted rectangular fins provide higher heat transfer rates. Aluminum plate enhances the heat transfer rate, and rectangular notched fins offer the highest heat transfer rates among all notch geometries.

Nomenclature

A – area, $\left[m^2\right]$
b – fin length $[m]$
Gr – Grashof number
h – coefficient of convection heat transfer $\left[W / m^2\,{}^{\circ}C\right]$
k – thermal conductivity $\left[W / m^{\circ}C\right]$
L – fin width $[m]$
Nu – Nusselt number
P – perimeter $[m]$
Pr – Prandtl number
q – rate of heat transfer $[W]$
R – thermal resistance $\left[{}^{\circ}C / W\right]$
Re – Reynolds number
S – the space between the fins $[m]$
T – temperature $\left[{}^{\circ}C\right]$
t – thickness of fin $[m]$
V – volume of fin $\left[m^3\right]$
W – width of heat sink $[m]$
x – x-coordinate
Δx – thickness of heat sink base plate $[m]$
β – volume coefficient of expansion $[1 / K]$
η – thermal efficiency
ν – the kinematic viscosity $\left[m^2 / s\right]$
∞ – ambient

Subscript symbols

$base$ – base plate
c – cross-section
f – fin
o – optimum
p – profile
t – total
unf – unfinned
w – wall

References

[1] Kordyban T. (1998): *Hot Air Rises and Heat Sinks: Everything You Know About Cooling Electronics Is Wrong.*– ASME Press, p.236, https://doi.org/10.1115/1.800741.

[2] Lee S. (1995): *Optimum design and selection of heat sinks*.– IEEE Transactions on Components, Packaging, and Manufacturing Technology: Part A, vol.18, No.4, pp. 812-817, doi: 10.1109/95.477468.

[3] Londhe K. and Kaushik V.R. (2017): *Heat sink design for optimal performance of compact electronic appliances - a review*.– Journal for Advanced Research in Applied Sciences, vol.4, No.5, pp.13-21.

[4] Kandasamy R., Wang X.Q. and Mujumdar A.S. (2008): *Transient cooling of electronics using phase change material (PCM)-based heat sinks*.– Appl. Therm. Eng., vol.28, No.8-9, pp.1047-1057, doi: 10.1016/J.APPLTHERMALENG.2007.06.010.

[5] Kays W.M. and London A.L. (2011): *Compact Heat Exchangers*.– Third Edition,

[6] https://celsiainc.com/technology/heat-sink-design/.

[7] Lee H.S. (2010): *Thermal Design: Heat Sinks, Thermoelectrics, Heat Pipes, Compact Heat Exchangers, and Solar Cells*.– John Wiley & Sons, Inc., doi: 10.1002/9780470949979.

[8] Holman J.P. (2010): *Heat Transfer*.– 10th ed., p.758, McGraw-Hill Education

[9] Kraus A.D., Aziz A. and Welty J. (2001): *Extended Surface Heat Transfer*.– A Wiley-Interscience Publication J.WILEY & SONS, Inc., doi: 10.1002/9780470172582.

[10] Cengel Y., Turner R. and Smith R. (2001): *Fundamentals of thermal-fluid sciences*.– Appl. Mech. Rev., vol.54, No.6, doi: 10.1115/1.1421126.

[11] Brown A. (1965): *Optimum dimensions of uniform annular fins*.– Int. J. Heat Mass Transf., vol.8, No.4, pp.655-662, doi: 10.1016/0017-9310(65)90051-7.

[12] Cobble M.H. (1971): *Optimum fin shape*.– J. Franklin Inst., vol.291, No.4, pp.283-292, doi: 10.1016/0016-0032(71)90184-0.

[13] Snider A.D. and Kraus A.D. (1987): *The quest for the optimum longitudinal fin profile*.– Heat Transfer Engineering, vol.8, No.2, doi: 10.1080/01457638708962790.

[14] Duffin R. (1959): *A variational problem relating to cooling fins*.– Indiana University Mathematics Journal, vol.8, No.1, doi: 10.1512/iumj.1959.8.58003.

[15] Maday C.J. (1974): *The minimum weight one-dimensional straight cooling fin*.– J. Eng. Ind., vol.96, No.1, pp.161-165, https://doi.org/10.1115/1.3438291 .

[16] Ernst E.R.G., Eckert R.G. and Drake R.M. (1987): *Analysis of heat and mass transfer*.– Hemisphere Pub. Corp., accessed: Nov. 21, 2023, [Online], available: https://inis.iaea.org/search/search.aspx?orig_q=RN:19100674

[17] Razelos P. and Imre K. (1983): *Minimum mass convective fins with variable heat transfer coefficients*.– J. Franklin Inst., vol.315, No.4, doi: 10.1016/0016-0032(83)90078-9.

[18] Aziz A. (1992): *Optimum dimensions of extended surfaces operating in a convective environment*.– Appl. Mech. Rev., vol.45, No.5, doi: 10.1115/1.3119754.

[19] Bergman T.L., Lavine A.S., Incropera F.P. and DeWitt D.P. (2015): *Fundamentals of Heat and Mass Transfer*.– John Wiley & Sons, USA.

[20] Chung B.T.F. and Iyer J.R. (1993): *Optimum design of longitudinal rectangular fins and cylindrical spines with variable heat transfer coefficient*.– Heat Transfer Engineering, vol.14, No.1, pp.31-41, doi: 10.1080/01457639308939792.

[21] Schmidt E. (1926): *Die Wärmeübertragung Durch Rippen*.

[22] Chung B.T.F. and Nguyen L.D. (1986): *Optimization of design parameters for radiating longitudinal fins of various geometries*.– in AIAA Paper, doi: 10.2514/6.1986-150.

[23] Levy E.K. (1971): *Optimum plate spacings for laminar natural convection heat transfer from parallel vertical isothermal flat plates*.– J. Heat Transfer, vol.93, No.4, pp.463-465, doi: 10.1115/1.3449847.

[24] Elenbaas W. (1942): *Heat dissipation of parallel plates by free convection*.– Physica, vol.9, No.1, pp.1-28, doi: 10.1016/S0031-8914(42)90053-3.

[25] Bodoia J.R. and Osterle J.F. (1962): *The development of free convection between heated vertical plates*.– J. Heat Transfer, vol.84, No.1, pp.40-43, doi: 10.1115/1.3684288.

[26] Bar-Cohen A. and Jelinek M. (1985): *Optimum arrays of longitudinal, rectangular fins in corrective heat transfer*.– Heat Transfer Engineering, vol.6, No.3, pp.68-78, doi: 10.1080/01457638508939633.

[27] Starner K.E. and McManus H.N. (1963): *An experimental investigation of free-convection heat transfer from rectangular-fin arrays*.– J. Heat Transfer, vol.85, No.3, pp.273-277, doi: 10.1115/1.3686097.

[28] Jones C.D. and Smith L.F. (1970): *Optimum arrangement of rectangular fins on horizontal surfaces for free-convection heat transfer*.– J. Heat Transfer, vol.92, No.1, doi: 10.1115/1.3449648.

[29] Barrett A.V. and Obinelo I.F. (1997): *Characterization of longitudinal fin heat sink thermal performance and flow bypass effects through CFD methods*.– in Annual IEEE Semiconductor Thermal Measurement and Management Symposium, doi: 10.1109/stherm.1997.566793.

[30] De Lieto Vollaro A., Grignaffini S. and Gugliermetti F. (1999): *Optimum design of vertical rectangular fin arrays*.– International Journal of Thermal Sciences, vol.38, No.6, pp. 525-529, doi: 10.1016/S1290-0729(99)80025-8.

[31] Baskaya S., Sivrioglu M. and Ozek M. (2000): *Parametric study of natural convection heat transfer from horizontal rectangular fin arrays*.– International Journal of Thermal Sciences, vol.39, No.8, pp.797-805, doi: 10.1016/S1290-0729(00)00271-4.

[32] Bar-Cohen A., Iyengar M. and Kraus A.D. (2003): *Design of optimum plate-fin natural convective heat sinks*.– Journal of Electronic Packaging, Transactions of the ASME, vol.125, No.2, pp.208-216, SPEC., doi: 10.1115/1.1568361.

[33] Yazicioğlu B. and Yüncü H. (2007): *Optimum fin spacing of rectangular fins on a vertical base in free convection heat transfer*.– Heat and Mass Transfer/Waerme- und Stoffuebertragung, vol.44, No.1, pp.11-21, doi: 10.1007/s00231-006-0207-6.

[34] Mittelman G., Dayan A., Dado-Turjeman K. and Ullmann A. (2007): *Laminar free convection underneath a downward facing inclined hot fin array*.– Int. J. Heat Mass Transf., vol.50, No.13-14, pp.2582-2589, doi: 10.1016/j.ijheatmasstransfer.2006.11.033.

[35] Shaeri M.R. and Yaghoubi M. (2009): *Thermal enhancement from heat sinks by using perforated fins*.– Energy Convers. Manag., vol.50, No.5, pp.1264-1270, doi: 10.1016/j.enconman.2009.01.021.

[36] Kim D.K. (2012): *Thermal optimization of plate-fin heat sinks with fins of variable thickness under natural convection*.– Int. J. Heat Mass Transf., vol.55, No.4, pp.752-761, doi: 10.1016/j.ijheatmasstransfer.2011.10.034.

[37] Ahmadi M., Mostafavi G. and Bahrami M. (2014): *Natural convection from rectangular interrupted fins*.– International Journal of Thermal Sciences, vol.82, No.1, pp.62-71, doi: 10.1016/j.ijthermalsci.2014.03.016.

[38] Pouryoussefi S. and Zhang Y. (2015): *Experimental study of air-cooled parallel plate fin heat sinks with and without circular pin fins between the plate fins*.– Journal of Applied Fluid Mechanics, vol.8, No.3, pp.515-520, doi: 10.18869/acadpub.jafm.67.222.22930.

[39] Karamanis G. and Hodes M. (2016): *Longitudinal-fin heat sink optimization capturing conjugate effects under fully developed conditions*.– J. Therm. Sci. Eng. Appl., vol.8, No.4, p.7, doi: 10.1115/1.4034339.

[40] Shadlaghani A., Tavakoli M.R., Farzaneh M. and Salimpour M.R. (2016): *Optimization of triangular fins with/without longitudinal perforate for thermal performance enhancement*.– Journal of Mechanical Science and Technology, vol.30, No.4, pp.1903-1910, doi: 10.1007/s12206-016-0349-5.

[41] Hong S.H. and Chung B.J. (2016): *Variations of the optimal fin spacing according to Prandtl number in natural convection*.– International Journal of Thermal Sciences, vol.101, pp.1-8, doi: 10.1016/J.IJTHERMALSCI.2015.10.026.

[42] Dogan M. and Dogan D. (2017): *Experimental investigation of natural convection heat transfer from fin arrays for different tip-to-base fin spacing ratios*.– Isi Bilimi Ve Teknigi Dergisi/ Journal of Thermal Science and Technology, vol.37, No.1, pp.147-157.

[43] Jaya Krishna D. (2018): *Operational time and melt fraction based optimization of a phase change material longitudinal fin heat sink*.– J. Therm. Sci. Eng. Appl., vol.10, No.6, p.4, doi: 10.1115/1.4040988.

[44] Hou D., Xin X. and Qian J. (2020): *Analysis of natural convection heat transfer from vertical and inclined plate fin heat sinks*.– in Lecture Notes in Electrical Engineering, Springer Verlag, pp.479-487, doi: 10.1007/978-981-32-9441-7_49.

[45] Jasim H.H. (2020): *Heat transfer enhancement from heat sources using optimal design of combined fins heat-sinks*.– Propulsion and Power Research, vol.9, No.4, pp.372-382, doi: 10.1016/j.jppr.2020.11.002.

[46] Xie L., Zhang Y., Ge M. and Zhao Y. (2022): *Topology optimization of heat sink based on variable density method*.– Energy Reports, vol.8, pp.718-726, doi: 10.1016/J.EGYR.2021.11.214.

[47] Obaid A.J. and Hameed V.M. (2023): *An experimental and numerical comparison study on a heat sink thermal performance with new fin configuration under mixed convective conditions*.– S. Afr. J. Chem. Eng., vol.44, pp.81-88, doi: 10.1016/J.SAJCE.2023.01.009.

[48] Kaya M. and Kaya Ş. (2023): *Optimum fin spacing of rectangular fins on aluminum heat sinks plates*.– Proceedings of the Romanian Academy Series A - Mathematics Physics Technical Sciences Information Science, vol.24, No.2, pp.157-157.

Received: February 15, 2024
Revised: August 7, 2024